Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

Find the Maclaurin series for the function $f(x) = x \cos x$.

Evaluate $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$ by using the Maclaurin series of e^x.

Find the sum of the series $\frac{1}{1\cdot 2}-\frac{1}{2\cdot 2^2}+\frac{1}{3\cdot 2^3}-\frac{1}{4\cdot 2^4}+\cdots$

Find the sum of the series $\dfrac{1}{1 \cdot 2} - \dfrac{1}{3 \cdot 2^3} + \dfrac{1}{5 \cdot 2^5} - \dfrac{1}{7 \cdot 2^7} + \cdots$

8.7 Taylor Series

Compute the Taylor series of $f(x) = \dfrac{1}{1-x}$ centered at 0 and find its interval of convergence.

List of power series (centered at 0) that you must memorize. "I" means Interval of Convergence.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad \text{I: } (-1, 1)$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad \text{I: } (-\infty, \infty)$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad \text{I: } (-\infty, \infty)$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad \text{I: } (-\infty, \infty)$$

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad \text{I: } [-1, 1]$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad \text{I: } (-1, 1]$$

8.6 When is a function equal to its Taylor series?

A function $f(x)$ is equal to its Taylor series only for x in the interval of convergence.

Given a function f, the Taylor series $T(x)$ is trying to approximate f as a polynomial, but because not every function is a polynomial, the cost of this conversion is that we need infinite sums and a restriction of our domain to the interval of convergence.

https://www.desmos.com/calculator/vtxtcw9y72

8.7 Taylor Series

The **Taylor series of** $f(x)$ **centered at** a is defined as

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

Dissecting the Notations

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Find the Taylor series for $f(x) = x^4 - 3x^2 + 1$ at $a = 1$.

8.7 Taylor Series

Represent $f(x) = \sin x$ as the sum of its Taylor series centered at $\frac{\pi}{3}$.

Find the 42nd derivative of $\sin(x^2)$ at $x = 0$.