So far we've learned how to approximate functions using their Taylor polynomials. For example,

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

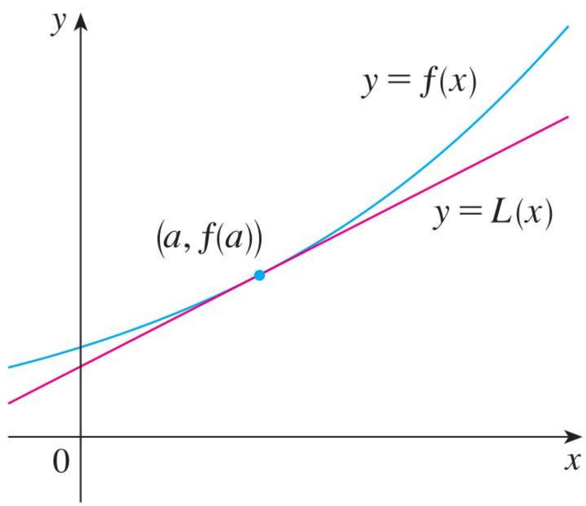
But this approximation requires knowing the derivatives of many orders at the center.

What if we only know a function's initial value and its first derivative? How should we draw a graph that approximates the original function?

In Calculus 1, we learned how to approximate functions using tangent lines which only uses information about a function's first derivative and an initial value.

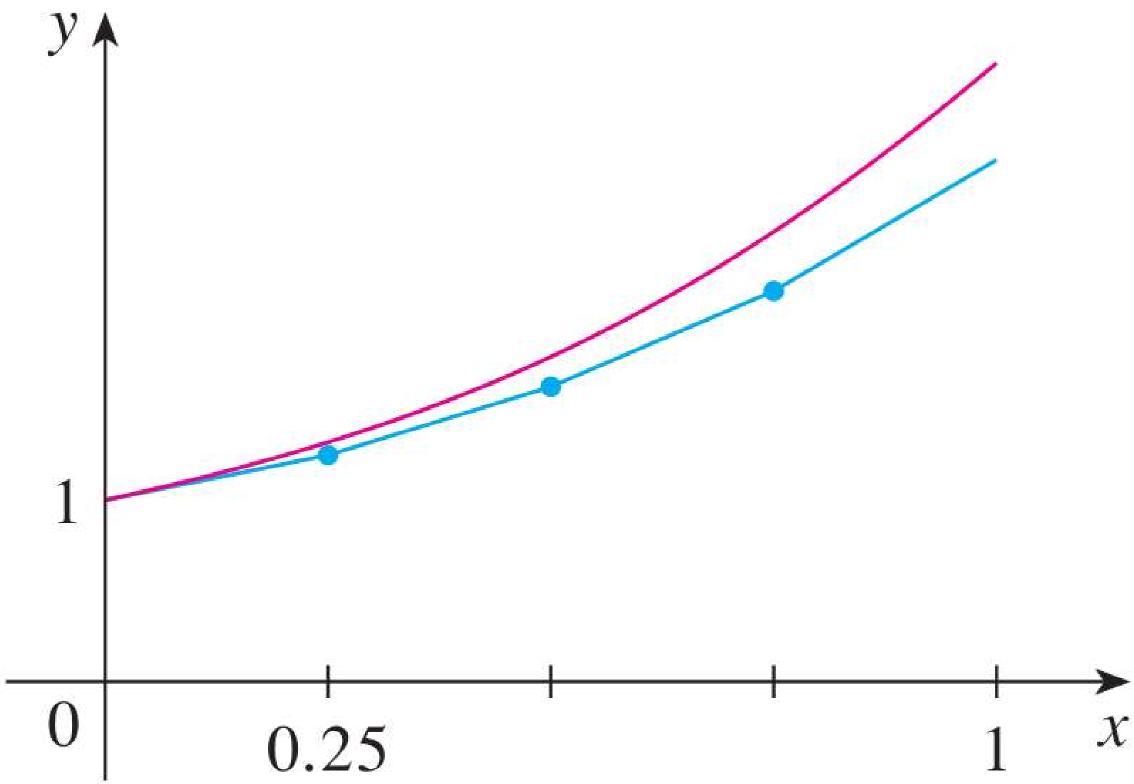
The equation of a tangent line at $(a, f(a))$:

$$L(x) = f'(a)(x - a) + f(a)$$



But this approximation is only good for a short distance from the base point. How can we extend the viability of straight-line approximations?

The simplest way is to just draw more straight lines. An approximation using straight line-segments is called the **Euler's Method**.



7.2 Euler's Method

Eulers method is the process of moving only a short distance along the original tangent line and then making a midcourse correction by changing direction as indicated by the slope field.

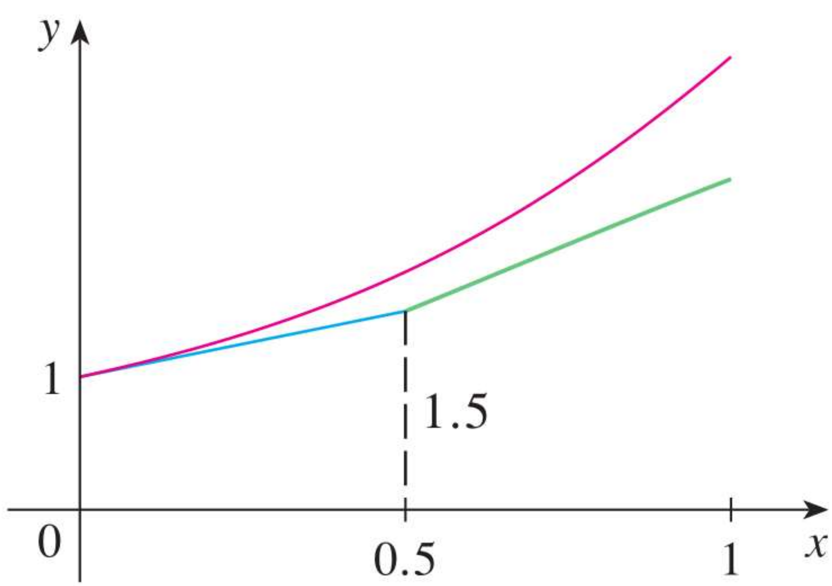


FIGURE 13

Euler approximation with step size 0.5

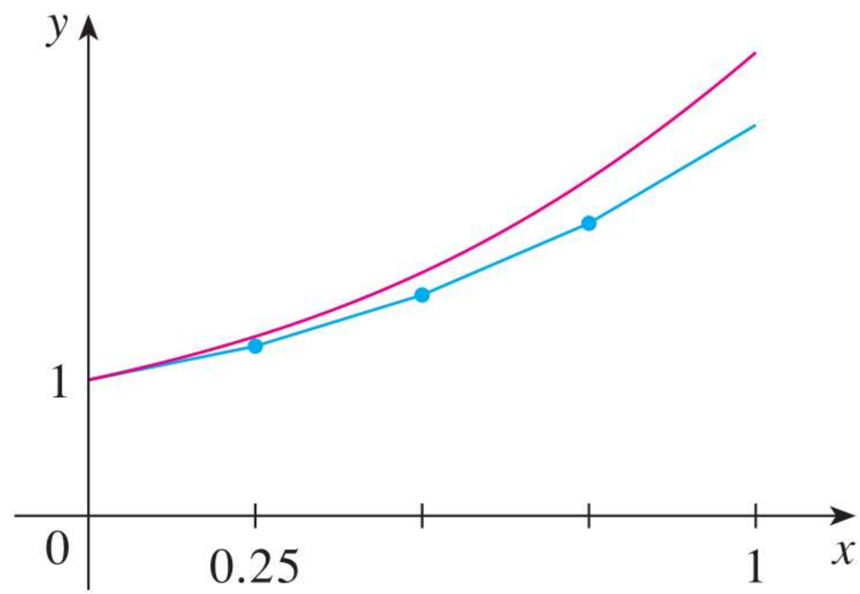


FIGURE 14

Euler approximation with step size 0.25

7.2 Euler's Method

Start at the point given by the initial value and move in the direction indicated by the slope field. Stop after a short time, look at the slope at the new location, and move in that direction. Repeat.

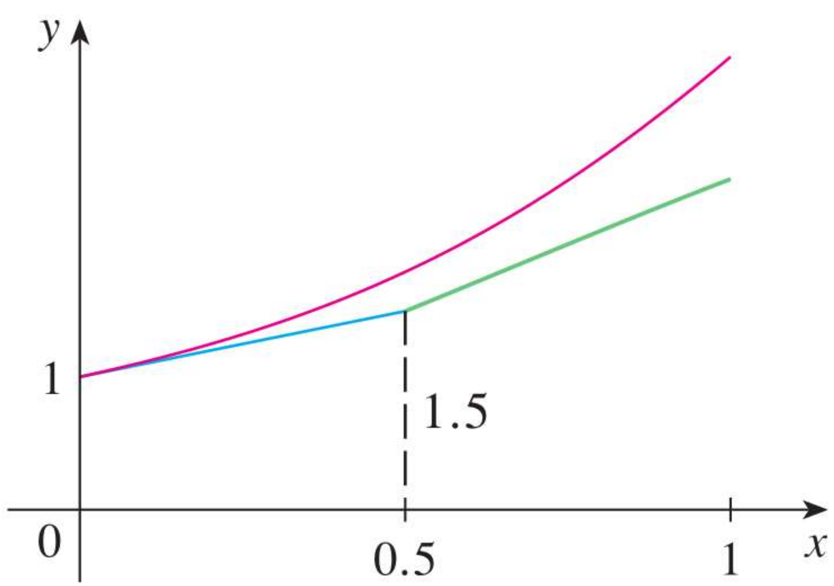


FIGURE 13

Euler approximation with step size 0.5

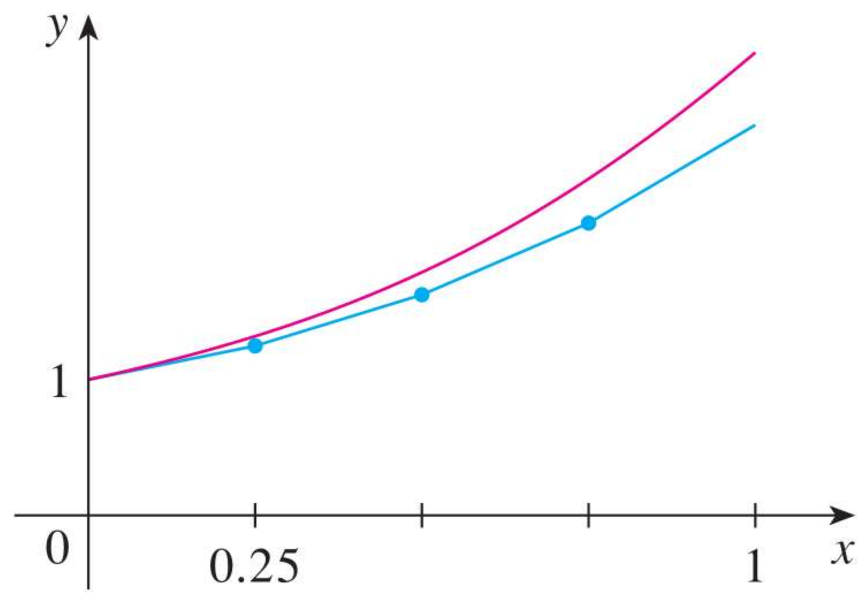


FIGURE 14

Euler approximation with step size 0.25

7.2 Euler's Method

- Euler's method does not produce the exact solution to an initial-value problem; it only gives an approximation.
- But by decreasing the step size (and therefore increasing the number of midcourse corrections), we obtain successively better approximations to the exact solution.

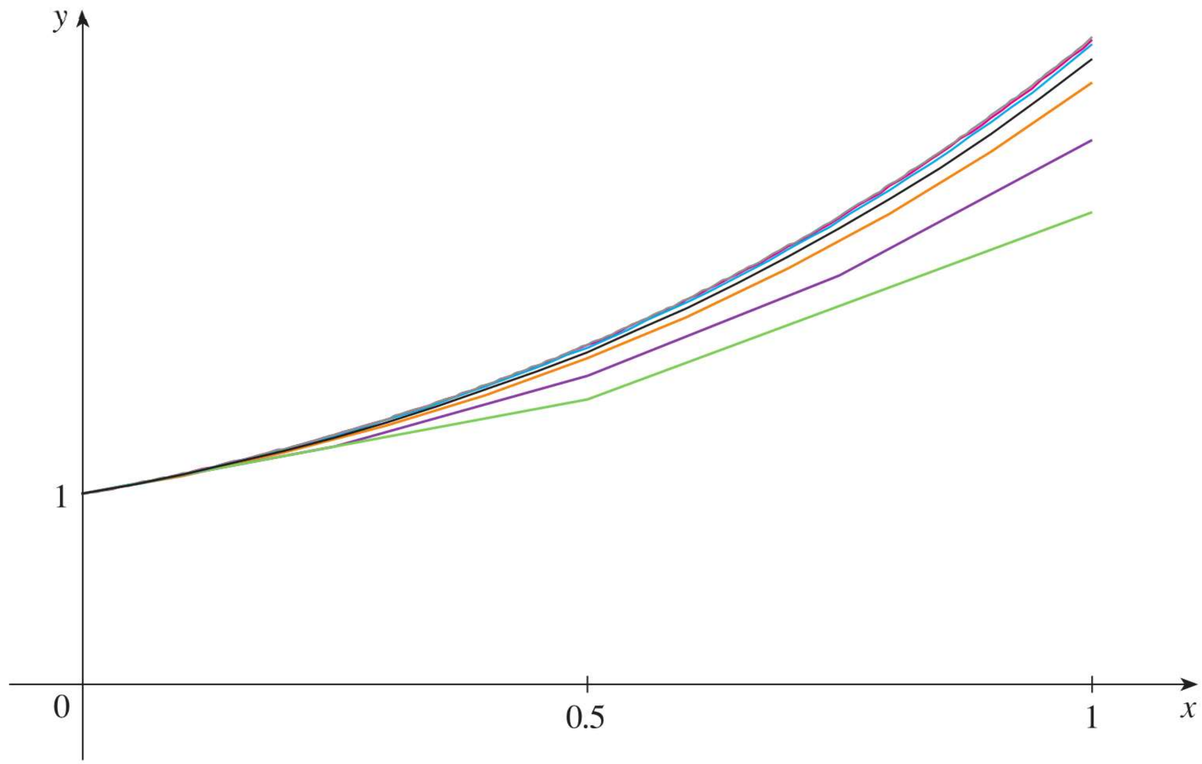


FIGURE 16
Euler approximations approaching the exact solution

Euler's Method (Computing the endpoints of the line segments). Approximate values for the solution of the initival-value problem y', $y(x_0) = y_0$, with step size Δx, at x_{n+1}, are

$$y_{n+1} = y'(x_n, y_n)\Delta x + y_n$$

next y-value   slope   current y-value

where

$$x_{n+1} = x_n + \Delta x$$

Note: y' is a function of both x and y so $y'(x_n, y_n)$ means we are plugging in x_n for x and y_n for y.

Here $y(x_0) = y_0$ is the initial value, meaning (x_0, y_0) is where the Euler's method starts. y' is the first derivative, which is used to compute the slopes of the line segments.

Use Euler's method with step size $\Delta x = 0.2$ to estimate $y(3)$, where $y(x)$ is the solution of the initial-value problem $y' = x + y$, $y(2) = 0$.

$y_{n+1} = y' \Delta x + y_n$

n	x_n	y_n	$y'(x_n, y_n)$	$y'\Delta x$	x_{n+1}	y_{n+1}
0	2	0	$2+0=2$	0.4	2.2	$0.4+0=0.4$
1	2.2	0.4	2.6	0.52	2.4	0.92
2	2.4	0.92	3.32	0.664	2.6	1.584
3	2.6	1.584	4.184	0.8368	2.8	2.4208
4	2.8	2.4208	5.2208	1.04416	3.0	3.46496
5	3.0	3.46496				

$y(3) \approx 3.46496$

7.2 Desmos Activity (Optional)

Hey, students!

Go to student.desmos.com
and type in:

7TB 26H