Reminder

- Handouts are due Friday.
- Check WebAssign for online homework.
- Written homework is due Thursday.
- Syllabus last page sign and return by Friday.

Daily Quiz

- Go to Socrative.com and complete the quiz.
- Use your full name.
- Room Name: HONG5824

5.6 Integration by Parts - Your choice matters

Integrating by parts Find $\int x \sin x \, dx$.

Recall that last time, we found that $\int x \sin x \; dx = -x\cos x + \sin x + C$. What if we choose a different u and dv?

Yesterday, we chose $u = x$ and $dv = \sin x \, dx$.

Today, let's try $u = \sin x$ and $dv = x\,dx$.

$u = \sin x$	$v = \frac{x^2}{2}$
$du = \cos x \, dx$	$dv = x\,dx$

$$\int u \, dv = uv - \int v \, du$$

$$\int x \sin x \, dx = \sin x \frac{x^2}{2} - \int \frac{x^2}{2} \cos x \, dx$$

$$= \sin x \frac{x^2}{2} - \frac{1}{2}\int x^2 \cos x \, dx$$

The new integrand $x^2 \cos x$ has a higher degree of x than we started with. This is no good because if we integrate by parts again on $x^2\cos x$, we'll get $\int x^3 \sin x \, dx$

Observation: Choice of u and dv matters a lot!

5.5 How to choose your U and dV: LIATE

Integration by parts requires us to know the derivative of u and the anti-derivative of dv. Since some functions are harder to integrate than others, we let dv be a function that is **easier to integrate** while we let u be the function that is **harder to integrate**.

Following this heuristic, we have a rule that helps us pick the right u and we let the remainder be dv: **LIATE**.

When choosing u, follow the below priority list.

1. **L**ogarithmic functions (e.g. $\log x$)
2. **I**nverse Trig functions (e.g. $\arctan x$)
3. **A**lgebraic functions (e.g. $x^2, \frac{1}{x^7}$)
4. **T**rig functions (e.g. $\tan x$)
5. **E**xponential functions (e.g. $2^x, e^x$)

5.6 Integration by Parts

Evaluate $\int \ln x \, dx$.

$u = \ln x$	$v = x$
$du = \frac{1}{x} dx$	$dv = dx$

$$uv - \int v\,du = x\ln x - \int x \frac{1}{x} dx$$

$$= x\ln x - x + C$$

Guidelines for selecting u and dv

LIATE

- L: Logs
- I: Inverse Trig
- A: algebraic functions
- T: Trig functions
- E: exponentials

Choose u to be the function that comes first in this list.

5.6 Integration by Parts

Integrating by parts twice Find $\int \underbrace{t^2}_{\text{algebraic}} \overbrace{e^t}^{\text{exponential}} \, dt.$

LIATE

algebraic

$u = t^2$	$v = e^t$
$du = 2t\,dt$	$dv = e^t\,dt$

$$uv - \int v\,du = t^2 e^t - \int e^t\, 2t\,dt$$

$$= t^2 e^t - 2 \underbrace{\int t e^t\,dt}_{I}$$

$I = \int t e^t\,dt$ (Integ by parts again)

algebraic (t), exponential (e^t)

LIATE

$u = t$	$v = e^t$
$du = dt$	$dv = e^t\,dt$

$$uv - \int v\,du = te^t - \int e^t\,dt$$

$$= te^t - e^t$$

$$\int t^2 e^t\,dt = t^2 e^t - 2\left[te^t - e^t\right] + C$$

What happens if you don't follow LIATE?

Integrating by parts twice Find $\int t^2 e^t\, dt$.

algebmic

Suppose we used LIATE the first time but not the second time.

$$\begin{array}{l|l} u = t^2 & v = e^t \\ \hline du = 2t\,dt & dv = e^t dt \end{array}$$

$$\int t^2 e^t dt = t^2 e^t - \int e^t 2t\, dt = t^2 e^t - 2\underbrace{\int t e^t dt}_{I}$$

$$I = \int t e^t dt$$

We are supposed to pick $u = t$ and $dv = e^t dt$ following LIATE.
What if we forgot? Let's try $u = e^t$ and $dv = t\,dt$.

$$\begin{array}{l|l} u = e^t & v = \frac{t^2}{2} \\ \hline du = e^t dt & dv = t \end{array}$$

$$\int t e^t dt = e^t \frac{t^2}{2} - \int \frac{t^2}{2} e^t dt = \frac{t^2 e^t}{2} - \frac{1}{2}\int t^2 e^t dt$$

Observe that we now have t^2 instead of t. This is bad because if we try to simplify, we get back to where we started, making no progress.

$$t^2 e^t - 2I = t^2 e^t - 2\left[\frac{t^2 e^t}{2} - \frac{1}{2}\int t^2 e^t dt\right] = t^2 e^t - t^2 e^t + \int t^2 e^t dt$$

$$= 0 + \int t^2 e^t dt$$

Conclusion:
Always follow LIATE

5.6 Integration by Parts (Substitution before By-Parts)

$$\int \cos\sqrt{x}\, dx$$

Simple substitution

$w = \sqrt{x}$

$dw = \frac{1}{2}x^{-\frac{1}{2}}dx$

$\sqrt{x}\, dw = \frac{1}{2}dx$

$2w\,dw = dx$

$$\int \cos(w)(2w\, dw)$$

$$= 2\int w\cos(w)\, dw$$

Now use integration by parts on the above integral.

LIATE

w is algebraic

$u = w$	$v = \sin(w)$
$du = dw$	$dv = \cos(w)dw$

$$\int w\cos(w) = uv - \int v\,du$$

$$= w\sin(w) - \int \sin(w)dw$$

$$= w\sin(w) + \cos(w)$$

Hence $\int \cos\sqrt{x}\,dx$

$$= 2\int w\cos(w)\,dw.$$

$$= 2\left[w\sin(w) + \cos(w)\right] + C$$

$$= 2\left[\sqrt{x}\sin(\sqrt{x}) + \cos(\sqrt{x})\right] + C$$

5.6 Integration by Parts (Definite Integrals)

$$\int_a^b f(x)g'(x)\,dx = f(x)g(x)\Big]_a^b - \int_a^b g(x)f'(x)\,dx$$

5.6 Integration by Parts

$$\int_0^1 \underbrace{\arctan(x)}_{\text{Inverse Trig}} \, dx$$

LIATE

$u = \arctan x$	$v = x$
$du = \dfrac{1}{1+x^2}\,dx$	$dv = dx$

$$uv\Big]_0^1 - \int_0^1 v\,du = x\arctan x\Big]_0^1 - \underbrace{\int_0^1 x\,\frac{1}{1+x^2}\,dx}_{I}$$

$$I = \int_0^1 \frac{x}{1+x^2}\,dx \quad \text{(simple u-sub)}$$

$u = 1+x^2$ $\quad u(0) = 1+0 = 1$

$du = 2x\,dx$ $\quad u(1) = 1+1^2 = 2$

$$\int_0^1 \frac{x}{1+x^2}\,dx = \int_{u(0)}^{u(1)} \frac{du/2}{u} = \frac{1}{2}\int_1^2 \frac{du}{u}$$

$$= \frac{1}{2}\ln|u|\Big]_1^2 = \frac{1}{2}\left[\ln(2) - \cancel{\ln 1}^{\,0}\right]$$

$$\int_0^1 \arctan(x)\,dx = x\arctan x\Big]_0^1 - I$$

$$= [\arctan 1 - 0] - \frac{1}{2}\ln(2)$$

$$= \frac{\pi}{4} - \frac{\ln(2)}{2}$$