Reminder

- Handouts are due Friday.
- Check WebAssign for online homework.
- Written homework is due Thursday.
- Syllabus last page sign and return by Friday.

Indefinite Integral Domino Chain

- Get in a group of 4 or 5 and start matching the top half of a domino with the bottom half of another domino.
- Split the work: 5-6 cards per person.
- You will need a scratch paper to work out the integrals.
- They should form a chain; when finished, they become a loop.
- You got 10 minutes.

Daily Quiz

- Go to Socrative.com and complete the quiz.
- Use your full name.
- Room Name: HONG5824

5.5 The Substitution Rule (Review)

Find $\int x^3 \cos(x^4 + 2)\, dx$.

We see x^3 and x^4+2. Since the derivative of x^4+2 is $4x^3$, choosing $u = x^4+2$ may work.

$u = x^4+2$

$du = 4x^3 dx$

$\frac{du}{4} = x^3 dx$

$$\int x^3 \cos(x^4+2)\,dx = \int \cos(x^4+2)\, x^3 dx$$
$$= \int \cos(u)\, \frac{du}{4}$$
$$= \frac{1}{4} \int \cos(u)\, du$$
$$= \frac{1}{4} \sin(u) + C$$
$$= \frac{1}{4} \sin(x^4+2) + C$$

5.5 The Substitution Rule (Review)

Find $\int \frac{x}{\sqrt{1-4x^2}}\,dx$.

We see x and $1-4x^2$. Since the derivative of $1-4x^2$ is $-8x$, choosing $u = 1-4x^2$ should work.

$u = 1-4x^2$

$du = -8x\,dx$

$\frac{du}{-8} = x\,dx$

$$\int \frac{x}{\sqrt{1-4x^2}}\,dx = \int \frac{x\,dx}{\sqrt{1-4x^2}}$$

$$= \int \frac{\frac{du}{-8}}{\sqrt{u}}$$

$$= -\frac{1}{8}\int \frac{du}{\sqrt{u}}$$

$$= -\frac{1}{8}\int u^{-\frac{1}{2}}\,du$$

$$= -\frac{1}{8}\,\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= -\frac{1}{8}\,\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= -\frac{1}{8}\cdot 2u^{\frac{1}{2}} + C$$

$$= -\frac{1}{4}\sqrt{u} + C$$

$$= -\frac{1}{4}\sqrt{1-4x^2} + C$$

5.5 The Substitution Rule (Review)

Two possible substitutions Evaluate $\int \sqrt{2x+1}\, dx$.

1. Let's first try $u=2x+1$.

$u = 2x+1$

$du = 2dx$

$\frac{du}{2} = dx$

$$\int \sqrt{2x+1}\, dx = \int \sqrt{u}\, \frac{du}{2}$$

$$= \frac{1}{2} \int \sqrt{u}\, du$$

$$= \frac{1}{2} \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{1}{2} \frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{1}{3} u^{3/2} + C$$

$$= \frac{1}{3} (2x+1)^{3/2} + C$$

5.5 The Substitution Rule (Review)

Two possible substitutions Evaluate $\int \sqrt{2x+1}\, dx$.

2. Let's try $u = \sqrt{2x+1}$.

$u = \sqrt{2x+1}$

$du = \frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot 2\,dx$

$du = (2x+1)^{-\frac{1}{2}}\,dx$

$\underbrace{(2x+1)^{\frac{1}{2}}}\,du = dx$

Observe that this is u

$u\,du = dx$

$$\int \sqrt{2x+1}\,dx = \int u \cdot u\,du$$

$$= \int u^2\,du$$

$$= \frac{u^3}{3} + C$$

$$= \frac{(\sqrt{2x+1})^3}{3} + C$$

Q: Is this the same answer as the previous answer?

5.5 Changing Boundary Values for the u-Substitution

5 **The Substitution Rule for Definite Integrals** If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

In other words, the x-bounds a and b change to the u-bounds u(a) and u(b).

5.5 Changing Boundaries

Evaluate $\int_0^4 \sqrt{2x+1}\,dx$

Recall that we've already computed the antiderivative of $\sqrt{2x+1}$.

$$\int \sqrt{2x+1}\,dx = \frac{(2x+1)^{3/2}}{3} + C$$

We can use this to evaluate $\int_0^4 \sqrt{2x+1}\,dx$ by plugging in the bounds 4 and 0.

$$\int_0^4 \sqrt{2x+1}\,dx = \left[\frac{(2x+1)^{3/2}}{3}\right]_0^4$$

$$= \frac{9^{3/2}}{3} - \frac{1^{3/2}}{3}$$

$$= \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$$

5.5 Changing Boundaries

Evaluate $\int_0^4 \sqrt{2x+1}\,dx$

Instead, we can also change the bounds right after we make our substitution $u = 2x+1$:

$\boxed{u = 2x+1}$

$du = 2dx$

$\frac{du}{2} = dx$

$$\int_{x=0}^{x=4} \sqrt{2x+1}\,dx = \int_{u=?}^{u=?} \sqrt{u}\,\frac{du}{2}$$

We compute the u-bounds by plugging in $x=4$ and $x=0$ to the substitution formula $u=2x+1$.

$u(4) = 2(4)+1 = 9$

$u(0) = 2(0)+1 = 1$

Hence the integral becomes

$$\int_{u(0)}^{u(4)} \sqrt{u}\,\frac{du}{2} = \int_1^9 \sqrt{u}\,\frac{du}{2}$$

$$= \frac{1}{2}\left[\frac{2}{3}\cdot u^{3/2}\right]_1^9$$

$$= \frac{1}{3}\left[u^{3/2}\right]_1^9$$

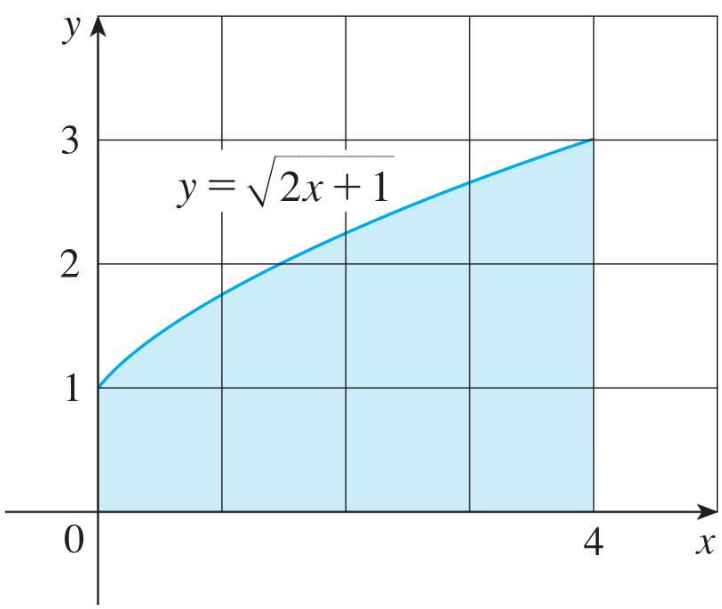
$$= \frac{1}{3}\left[9^{3/2} - 1^{3/2}\right]$$

$$= \frac{26}{3}$$

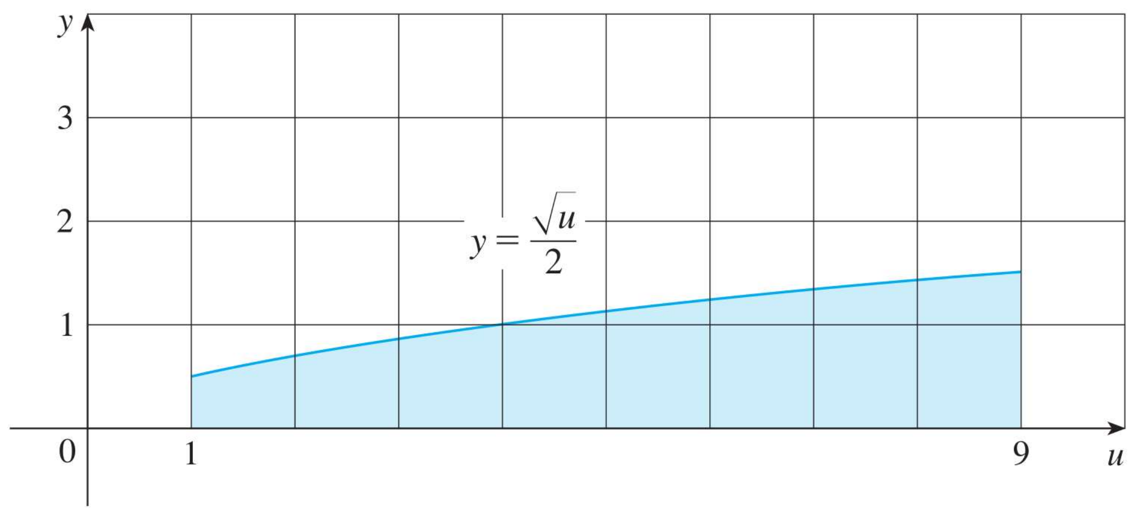
5.5 Visualizing the u-Substitution

When we change a variable, we shrink or stretch the region of integration based of the relation between du and dx.

$$\int_0^4 \sqrt{2x+1}\, dx$$



$$\int_1^9 \frac{\sqrt{u}}{2}\, du$$



5.5 Changing Boundaries

Evaluate $\displaystyle\int_1^2 \frac{dx}{(3-5x)^2}$.

Let $u = 3-5x$.

$u = 3-5x$

$du = -5\,dx$

$\frac{du}{-5} = dx$

Bounds

$u(2) = 3-5(2)$
$= -7$

$u(1) = 3-5(1)$
$= -2$

$$\int_1^2 \frac{dx}{(3-5x)^2} = \int_{-2}^{-7} \frac{du/_{-5}}{u^2}$$

$$= -\frac{1}{5}\int_{-2}^{-7} \frac{du}{u^2}$$

$$= -\frac{1}{5}\int_{-2}^{-7} u^{-2}du$$

$$= -\frac{1}{5}\left[\frac{u^{-2+1}}{-2+1}\right]_{-2}^{-7}$$

$$= \frac{1}{5}\left[u^{-1}\right]_{-2}^{-7}$$

$$= \frac{1}{5}\left[\frac{1}{-7} - \frac{1}{-2}\right]$$

$$= \frac{1}{5}\left[-\frac{1}{7} + \frac{1}{2}\right]$$

$$= \frac{1}{5}\left[-\frac{2}{14} + \frac{7}{14}\right]$$

$$= \frac{1}{5}\left(\frac{5}{14}\right)$$

$$= \frac{1}{14}$$

5.5 Changing Boundaries

Calculate $\int_1^e \frac{\ln x}{x}\,dx$.

Observe that the derivative of $\ln x$ is $\frac{1}{x}$ so we let $u = \ln x$.

$u = \ln x$

$du = \frac{1}{x}dx$

Bounds

$u(e) = \ln e = 1$

$u(1) = \ln 1 = 0$

$$\int_1^e \frac{\ln x}{x}dx = \int_1^e \ln x \, \frac{1}{x}dx$$

$$= \int_0^1 u\,du$$

$$= \left[\frac{u^2}{2}\right]_0^1$$

$$= \frac{1}{2} - \frac{0}{2} = \frac{1}{2}$$

5.6 Integration by Parts Is Product Rule in Reverse

Recall: Product Rule

Let $f(x)$ and $g(x)$ be differentiable functions.

then

$$\frac{d}{dx}(fg) = f'g + fg'$$

(Product Rule)

If we integrate both sides with respect to x, we get

$$\int \frac{d}{dx}(fg)\,dx = \int (f'g + fg')\,dx$$

$$fg = \int f'g\,dx + \int fg'\,dx$$

So

$$\int fg'\,dx = fg - \int f'g\,dx. \quad \text{(Integration by Parts)}$$

In another notation, we let $u = f$, $v = g$, $du = f'$, and $dv = g'$ and get $\boxed{\int u\,dv = uv - \int v\,du}$

5.6 Integration by Parts

$$\boxed{\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx}$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

5.6 Integration by Parts

Integrating by parts Find $\int x \sin x \, dx$.

To use integration by parts, we have to pick and choose our "u" and "dv".

Let's try $u = x$ and $dv = \sin x \, dx$.

$u = x$	$v = -\cos x$
$du = dx$	$dv = \sin x \, dx$

$$\int u \, dv = uv - \int v \, du$$

$$\int x \sin x \, dx = x(-\cos x) - \int -\cos x \, dx$$

$$= -x\cos x + \int \cos x \, dx$$

$$= -x\cos x + \sin x + C$$

Summary

- More u-sub
- Definite integrals with u-sub
- Intro to integration by parts