

Math 2300 Calculus 2, Spring 2017
Quiz 2

1. Find the following integrals:

(a)

$$\int \sec^2(x) \tan^6(x) dx = \int \frac{\sec^2 x u^6 du}{\sec^2 x}$$

$$u = \tan x$$

$$\frac{du}{dx} = \sec^2 x$$

$$= \frac{u^7}{7} + C = \frac{\tan^7 x}{7} + C$$

(b)

$$\int \sin^3(x) \cos^2(x) dx$$

$$\sin^3 x \cos^2 x = \sin^2 x \cos^2 x - \sin x$$

$$= (1 - \cos^2 x) \cos^2 x - \sin x$$

$$\int \sin^3 x \cos^2 x dx = \int (1 - \cos^2 x) \cos^2 x \sin x dx$$

$$\frac{du}{dx} = -\sin x$$

$$= -\int (1 - u^2) u^2 du = -\left(\int u^2 - u^4 du \right)$$

$$\frac{du}{-\sin x} = dx$$

$$= -\left(\frac{u^3}{3} - \frac{u^5}{5} \right) + C = -\left(\frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} \right) + C$$

2.

$$\int \frac{1}{\sqrt{4-x^2}} dx = \int \frac{1}{\sqrt{4-(2\sin\theta)^2}} \cdot 2\cos\theta d\theta$$

$$x = 2\sin\theta \quad \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\frac{dx}{d\theta} = 2\cos\theta$$

$$\begin{aligned} &= \int \frac{2\cos\theta}{\sqrt{4(1-\sin^2\theta)}} d\theta = \int \frac{\cancel{2}\cos\theta}{\cancel{2}\cos\theta} d\theta = \theta + C \\ &= \arcsin\left(\frac{x}{2}\right) + C \end{aligned}$$

3.

$$\int \frac{1}{16+x^2} dx = \int \frac{1}{16+16\tan^2\theta} \cdot \frac{4}{\cos^2\theta} d\theta$$

$$x = 4\tan\theta \quad \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\frac{dx}{d\theta} = 4\sec^2\theta = \frac{4}{\cos^2\theta}$$

$$\begin{aligned} &= \frac{1}{4} \int \frac{1}{(1+\tan^2\theta)\cos^2\theta} d\theta = \frac{1}{4} \int 1 d\theta \\ &= \frac{1}{4} \theta + C \\ &= \frac{1}{4} \arctan\left(\frac{x}{4}\right) + C \end{aligned}$$