1. $y = x^3 - x^2 = x^2(x-1)$. ZERO AT $x=1$, DOUBLE-ROOT AT $x=0$

INTERSECTION OF $y = x^3 - x^2$ AND $y = 2x$

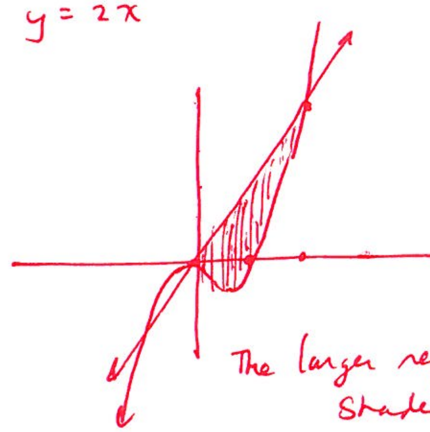
$x^3 - x^2 = 2x$

$x^3 - x^2 - 2x = 0$

$x(x^2 - x - 2) = 0$

$x(x-2)(x+1) = 0$

$x = 0,\ x = 2,\ x = -1$



The larger region is shaded.

$$\text{Area} = \int_0^2 2x - (x^3 - x^2)\,dx$$

$$= \int_0^2 2x - x^3 + x^2\,dx$$

$$= \left(x^2 - \frac{x^4}{4} + \frac{x^3}{3}\right)\Big|_0^2 = 4 - 4 + \frac{8}{3} = \frac{8}{3}$$

2. $x = y^2 - 4y = y(y-4)$ has y-intercepts at $y=0$, $y=4$ and it is a parabola, opening to the right.

$x = 2y - y^2 = y(2-y)$ has y-intercepts at $y=0$ and $y=2$ and it is a parabola opening to the left

intersections:

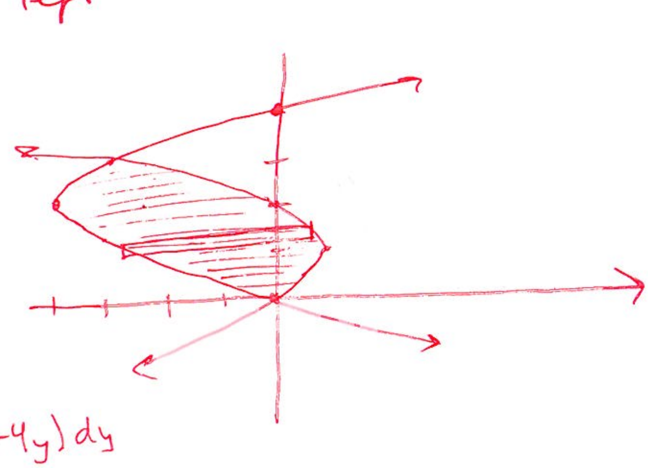
$y^2 - 4y = 2y - y^2$

$2y^2 = 6y$

$2y^2 - 6y = 0$

$2y(y-3) = 0$

$(0,0)$ and $(-3,3)$

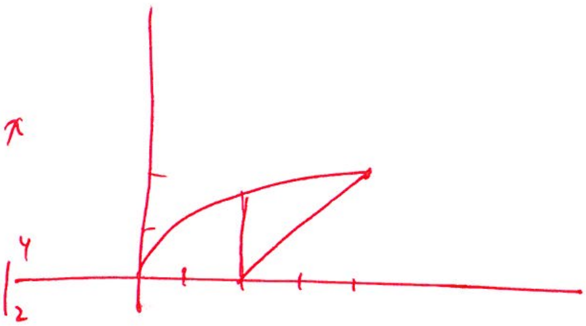


$$\text{area} = \int_{y=0}^{y=3} \underbrace{2y - y^2}_{\text{right}} - \underbrace{(y^2 - 4y)}_{\text{left}}\,dy$$

$$= \int_0^3 6y - 2y^2\,dy = \left(3y^2 - \frac{2}{3}y^3\right)\Big|_0^3 = 27 - 18 = 9$$

3.

split into 2 regions.



$$\text{Area} = \int_0^2 \sqrt{x}\,dx + \int_2^4 \sqrt{x} - (x-2)\,dx$$

$$= \frac{2}{3}x^{\frac{3}{2}}\Big|_0^2 + \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2} + 2x\right)\Big|_2^4$$

$$= \frac{2}{3}\cdot 2^{\frac{3}{2}} + \frac{2}{3}\cdot 8 - 8 + 8 - \left(\frac{2}{3}\cdot 2^{\frac{3}{2}} - 2 + 4\right)$$

$$= \frac{16}{3} - 2 = \frac{10}{3}$$

(yeay, same as when we integrated with respect to y)

4.

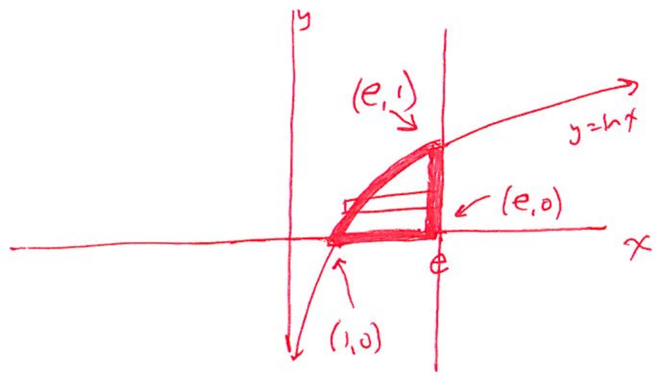
$y = \ln x, \quad y = 0, \quad x = e$

slice horizontally

solve for x:

$x = e$

$x = e^y$



$$\text{area} = \int_{y=0}^{y=1} e - e^y\,dy$$

$$= (ey - e^y)\Big|_0^1$$

$$= e - e - (0-1) = 1$$

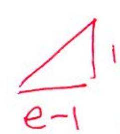
matches my estimate.

intersection:

$\begin{cases} x = e \\ x = e^y \end{cases} \Rightarrow e = e^y \Rightarrow y = 1$

$x = e$

Estimate: slightly larger than a triangle



so slightly larger than

$$\frac{e-1}{2} \approx \frac{1.7}{2} \approx .85$$