1. x is a function of y.

If it is a polynomial,

x has even degree,

negative leading coefficient

(look at end behavior)

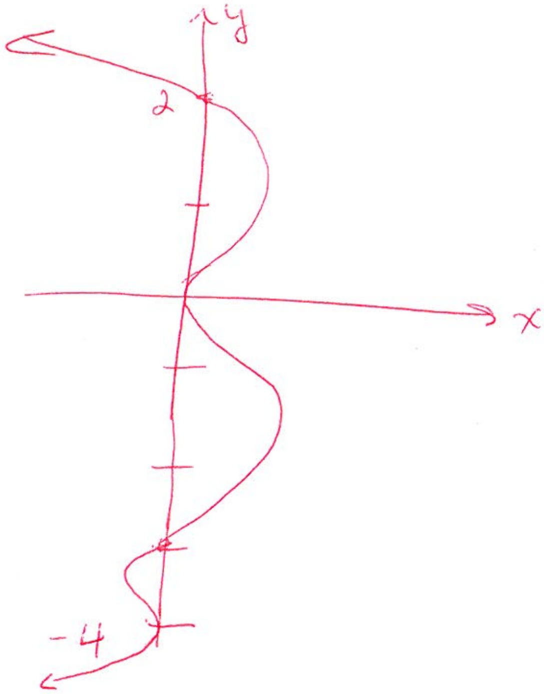
y-intercepts at $y=2$ (multiplicity 1)

at $y=0$ (bounce, so multiplicity 2)

$y=-3$ (cross, so multiplicity 1)

$y=-4$ (bounce, so multiplicity 2)

$$x = -y^2 \cdot (y-2)(y+3)(y+4)^2$$



2. $y = x^2 - x^4 = -(x^4 - x^2)$

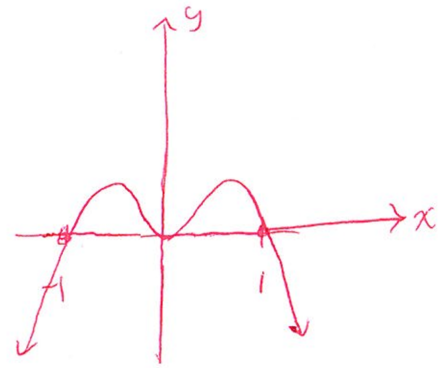
$= -x^2(x^2-1)$

$= -x^2(x+1)(x-1)$

even degree, negative leading coefficient

x-intercepts: $x=1$, $x=-1$, $x=0$

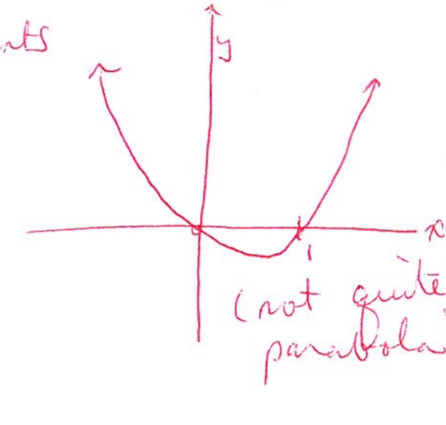
crosses crosses bounces



3. $y = x^4 - x = x(x^3-1) = x(x-1)(x^2+x+1)$ (because x^3-1 is the difference of two cubes)

$x=0$ and $x=1$ are x-intercepts.

even degree, positive leading coefficients



(not quite a parabola)

4. $y = x^5 + 2x^4 - x^3 - 2x^2$

$= x^2(x^3 + 2x^2 - x - 2)$

possible rational roots are $\pm 1, \pm 2$.

I try $x=1$ & find $y=0$.

So $x-1$ is a factor of (x^3+2x^2-x-2)

Long division gives other factor:

$$
\begin{array}{r}
x^2 + 3x + 2 \\
x-1 \,\big)\, \overline{x^3 + 2x^2 - x - 2} \\
-(x^3 - x^2) \\
3x^2 - x - 2 \\
-(3x^2 - 3x) \\
\hline
+2x - 2 \\
-(2x - 2) \\
\hline
0
\end{array}
$$

So $y = x^2(x-1)(x^2+3x+2)$

continue factoring:

$y = x^2(x-1)(x+2)(x+1)$ whew!

(you also could have used "factor by grouping to factor x^3+2x^2-x-2)

x-intercepts: $x=0$ (double), $x=1$, $x=-1$, $x=-2$