1. $\int \frac{1}{x^2+4x+29}\,dx$

$= \int \frac{1}{(x+2)^2+25}\,dx$

substitute: $\begin{cases} u = x+2 \\ du = dx \end{cases}$

$= \int \frac{1}{u^2+25}\,du$

$= \frac{1}{5}\arctan\frac{u}{5} + C$

$= \frac{1}{5}\arctan\frac{(x+2)}{5} + C$

complete the square:

$x^2+4x+29$

$= (x^2+4x+4) + 29 - 4$

$= (x+2)^2 + 25$

2. $\int \frac{1}{x^2-5x+18}\,dx$

$= \int \frac{1}{(x-\frac{5}{2})^2+\frac{47}{4}}\,dx$

$\begin{cases} u = x-\frac{5}{2} \\ du = dx \end{cases}$

$= \int \frac{1}{u^2+\frac{47}{4}}\,dx \qquad a = \frac{\sqrt{47}}{2}$

$= \frac{2}{\sqrt{47}}\arctan\frac{2(u)}{\sqrt{47}} + C$

$= \frac{2}{\sqrt{47}}\arctan\frac{2(x-\frac{5}{2})}{\sqrt{47}} + C$

complete the square:

$x^2-5x+18$

$= (x-5x+\frac{25}{4}) + 18 - \frac{25}{4}$

$= (x-\frac{5}{2})^2 + \frac{47}{4}$

3. $$\int \frac{dx}{4x^2-12x+58} = \frac{1}{4}\int \frac{dx}{x^2-3x+\frac{58}{4}}$$

$$= \frac{1}{4}\int \frac{dx}{\left(x-\frac{3}{2}\right)^2+\frac{58}{4}-\frac{9}{4}}$$

$$= \frac{1}{4}\int \frac{dx}{\left(x-\frac{3}{2}\right)^2+\frac{49}{4}}$$

$$= \frac{1}{4}\cdot\frac{2}{7}\arctan\frac{2\left(x-\frac{3}{2}\right)}{7}+C = \frac{1}{14}\arctan\left(\frac{2x-3}{7}\right)+C$$

4. $$\int \frac{dx}{\sqrt{11-10x-x^2}}$$

$$= \int \frac{dx}{\sqrt{11-(x^2+10x)}}$$

$$= \int \frac{dx}{\sqrt{11-(x^2+10x+25)+25}}$$

$$= \int \frac{dx}{\sqrt{36-(x+5)^2}} \qquad \begin{cases} u = x+5 \\ du = dx \end{cases}$$

$$= \arcsin\left(\frac{x+5}{6}\right)+C$$