

$$\begin{aligned}
 1. \int_0^1 x \sqrt{1-x^2} dx \\
 &= -\frac{1}{2} \int_1^0 \sqrt{u} du \\
 &= -\frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_1^0 \\
 &= \frac{1}{3}
 \end{aligned}$$

$$\begin{cases}
 u = 1-x^2 \\
 du = -2x dx \\
 -\frac{1}{2} du = x dx \\
 \text{when } x=0, u=1 \\
 \text{when } x=1, u=0
 \end{cases}$$

$$\begin{aligned}
 2. \int_e^{e^8} \frac{\sqrt[3]{\ln x}}{x} dx \\
 &= \int_1^8 u^{\frac{1}{3}} du \\
 &= \frac{3}{4} u^{\frac{4}{3}} \Big|_1^8 = 12 - \frac{3}{4} = 11.25
 \end{aligned}$$

$$\begin{cases}
 u = \ln x \\
 du = \frac{1}{x} dx \\
 \text{when } x=e, u=1 \\
 \text{when } x=e^8, u=8
 \end{cases}$$

$$\begin{aligned}
 3. \int \frac{\sec \sqrt{x}}{\sqrt{x}} dx \\
 &= 2 \int \sec u du
 \end{aligned}$$

$$\begin{cases}
 u = \sqrt{x} \\
 du = \frac{1}{2\sqrt{x}} dx \\
 2 du = \frac{1}{\sqrt{x}} dx
 \end{cases}$$

$$\begin{aligned}
 &= 2 \ln |\sec u + \tan u| + C \\
 &= 2 \ln |\sec \sqrt{x} + \tan \sqrt{x}| + C
 \end{aligned}$$

$$4. \int_0^{\sqrt[3]{\frac{1}{2}}} \frac{x^2}{\sqrt{1-x^6}} dx$$

$$\begin{aligned}
 &= \frac{1}{3} \int_0^{\frac{1}{2}} \frac{du}{\sqrt{1-u^2}} \\
 &= \frac{1}{3} \arcsin u \Big|_0^{\frac{1}{2}}
 \end{aligned}$$

$$\begin{cases}
 u = x^3 \\
 du = 3x^2 dx \\
 \frac{1}{3} du = x^2 dx \\
 \text{when } x=0, u=0 \\
 \text{when } x=\sqrt[3]{\frac{1}{2}}, u=\frac{1}{2}
 \end{cases}$$

$$= \frac{1}{3} \arcsin \left(\frac{1}{2}\right) = \frac{1}{3} \cdot \frac{\pi}{6} = \frac{\pi}{18}$$

$$\begin{aligned}
 5. \int \frac{3}{2x+1} dx \\
 &= \frac{1}{2} \int \frac{3}{u} du
 \end{aligned}$$

$$\begin{cases}
 u = 2x+1 \\
 du = 2 dx \\
 \frac{1}{2} du = dx
 \end{cases}$$

$$\begin{aligned}
 &= \frac{3}{2} \ln |u| + C \\
 &= \frac{3}{2} \ln |2x+1| + C
 \end{aligned}$$