1. $\int_0^1 x\sqrt{1-x^2}\,dx$

$= -\frac{1}{2}\int_1^0 \sqrt{u}\,du$

$= -\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}}\Big|_1^0$

$= \frac{1}{3}$

$\begin{cases} u = 1-x^2 \\ du = -2x\,dx \\ -\frac{1}{2}du = x\,dx \\ \text{when } x=0 \quad u=1 \\ \text{when } x=1, \quad u=0 \end{cases}$

2. $\int_e^{e^8} \frac{\sqrt[3]{\ln x}}{x}\,dx$

$= \int_1^8 u^{\frac{1}{3}}\,du$

$= \frac{3}{4}u^{\frac{4}{3}}\Big|_1^8 = 12 - \frac{3}{4} = 11.25$

$\begin{cases} u = \ln x \\ du = \frac{1}{x}dx \\ \text{when } x=e, \ u=1 \\ \text{when } x=e^8, \ u=8 \end{cases}$

3. $\int \frac{\sec\sqrt{x}}{\sqrt{x}}\,dx$

$= 2\int \sec u\,du$

$= 2\ln|\sec u + \tan u| + C$

$= 2\ln|\sec\sqrt{x} + \tan\sqrt{x}| + C$

$\begin{cases} u = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}}dx \\ 2du = \frac{1}{\sqrt{x}}dx \end{cases}$

4. $\int_0^{\sqrt[3]{1/2}} \frac{x^2}{\sqrt{1-x^6}}\,dx$

$= \frac{1}{3}\int_0^{\frac{1}{2}} \frac{du}{\sqrt{1-u^2}}$

$= \frac{1}{3}\arcsin u\Big|_0^{\frac{1}{2}}$

$= \frac{1}{3}\arcsin\left(\frac{1}{2}\right) = \frac{1}{3}\cdot\frac{\pi}{6} = \frac{\pi}{18}$

$u = x^3$
$du = 3x^2dx$
$\frac{1}{3}du = x^2dx$
when $x=0$, $u=0$
when $x=\sqrt[3]{\frac{1}{2}}$ $u=\frac{1}{2}$

5. $\int \frac{3}{2x+1}\,dx$

$= \frac{1}{2}\int \frac{3}{u}\,du$

$= \frac{3}{2}\ln|u| + C$

$= \frac{3}{2}\ln|2x+1| + C$

$\begin{cases} u = 2x+1 \\ du = 2dx \\ \frac{1}{2}du = dx \end{cases}$