CALC 2 - REVIEW/PREVIEW UNIT 4

u/du SUBSTITUTION (UNDOES THE CHAIN RULE)

EX 1 $\int x^2 \sec^2(x^3)\,dx$

SUBSTITUTION
$u = x^3$
$du = 3x^2\,dx$
$\frac{1}{3}du = x^2\,dx$

$= \int \underbrace{\sec^2(x^3)}_{=\sec^2 u}\ \underbrace{x^2\,dx}_{=\frac{1}{3}du}$

$= \frac{1}{3}\int \sec^2 u\,du$

$= \frac{1}{3}\tan(u) + C$ ⟩ integrate with u as the variable and don't forget to substitute back

$= \frac{1}{3}\tan(x^3) + C$ check: $\frac{d}{dx}\left(\frac{1}{3}\tan(x^3)\right) = \frac{1}{3}\cdot\sec^2(x^3)\cdot 3x^2 = x^2\sec^2(x^3)$ yay!

EX 2 A DEFINITE INTEGRAL:

$\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\,dx$

$\begin{cases} u = \frac{1}{x} \\ du = -\frac{1}{x^2}dx \\ \text{when } x=1,\ u=1 \\ \text{when } x=2,\ u=\frac{1}{2} \end{cases}$

$= -\int_1^2 \underbrace{e^{\frac{1}{x}}}_{e^u} \cdot \underbrace{-\frac{1}{x^2}dx}_{du}$

$= -\int_1^{\frac{1}{2}} e^u\,du = -e^u\Big|_1^{\frac{1}{2}} = -e^{\frac{1}{2}} - (-e) = e - \sqrt{e}$

DO THESE:

1. $\int_0^1 x\sqrt{1-x^2}\,dx$

2. $\int_e^{e^8} \frac{\sqrt[3]{\ln x}}{x}\,dx$

3. $\int \frac{\sec(\sqrt{x})}{\sqrt{x}}\,dx$

4. $\int_0^{\sqrt[3]{\frac{1}{2}}} \frac{x^2}{\sqrt{1-x^6}}\,dx$

5. $\int \frac{3}{2x+1}\,dx$