CALC 2 - REVIEW/PREVIEW UNIT 3

ALGEBRA AND INTEGRALS

REMEMBER THAT IT IS OFTEN HELPFUL TO SIMPLIFY AN INTEGRAND BEFORE TRYING TO ANTI-DIFFERENTIATE.

EXAMPLE 1: DISTRIBUTE

$$\int \sqrt{x}(x-1)\,dx$$
$$= \int x^{\frac{1}{2}}(x-1)\,dx$$
$$= \int x^{3/2} - x^{1/2}\,dx$$
$$= \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$$

EXAMPLE 2: EXPAND

$$\int (x^2+1)^4\,dx$$
$$= \int x^8 + 4x^6 + 6x^4 + 4x^2 + 1\,dx$$
$$= \frac{x^9}{9} + \frac{4}{7}x^7 + \frac{6}{5}x^5 + \frac{4}{3}x^3 + x + C$$

PASCAL'S TRIANGLE:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

These are the coefficients for the 4th degree binomial expansion

EXAMPLE 3: DISTRIBUTE

$$\int \frac{\sqrt{x}+1}{x}\,dx = \int x^{-\frac{1}{2}} + \frac{1}{x}\,dx$$
$$= 2x^{\frac{1}{2}} + \ln|x| + C$$

EXAMPLE 4: FACTOR/CANCEL

$$\int \frac{x^3+8}{x+2}\,dx$$

numerator is the sum of two cubes

$$= \int \frac{(x+2)(x^2-2x+4)}{x+2}\,dx$$
$$= \int x^2 - 2x + 4\,dx$$
$$= \frac{x^3}{3} - x^2 + 4x + C$$

EXAMPLE 5: YOU FIGURE OUT THE TRICK!

$$\int \frac{x-4}{\sqrt{x}+2}\,dx$$

multiply and divide by conjugate

$$= \int \frac{(x-4)(\sqrt{x}-2)}{(\sqrt{x}+2)(\sqrt{x}-2)}\,dx$$
$$= \int \frac{(x-4)(\sqrt{x}-2)}{x-4}\,dx$$
$$= \int \sqrt{x} - 2\,dx = \frac{2}{3}x^{\frac{3}{2}} - 2x + C$$

ALTERNATE TECHNIQUE: FACTOR NUMERATOR:

$$\int \frac{x-4}{\sqrt{x}+2}\,dx = \int \frac{(\sqrt{x}+2)(\sqrt{x}-2)}{\sqrt{x}+2}\,dx = \int \sqrt{x} - 2\,dx$$