WARMUP #10, SOLUTIONS

I A. $\lim\limits_{x \to \infty} \frac{e^x}{x^2} = \infty$ (e^x "beats" x^2)

B. $\lim\limits_{x\to\infty} \ln\left(\frac{x}{x+1}\right) = 0$ ($\frac{x}{x+1}$ approches 1 for large x, $\ln 1 = 0$)

C. $\lim\limits_{x\to\infty} e^{\frac{1-x^3}{x}} = 0$ ($\frac{1-x^3}{x}$ approches $-\infty$ for large x since x^3 "beats" x. $e^{-\text{large } \#}$ approaches 0)

D. $\lim\limits_{x\to\infty} \ln\left(\frac{3x}{x^2+1}\right) = -\infty$ ($\frac{3x}{x^2+1}$ approaches 0 for large x, ln(small positive #) approches $-\infty$.

E. $\lim\limits_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\arctan x} = 0$ ($\sin\left(\frac{1}{\text{large } \#}\right)$ = sin(small number), approaches 0. $\arctan x$ approches $\frac{\pi}{2}$. $\frac{0}{\pi/2} = 0$)

F. $\lim\limits_{x\to\infty} \frac{\ln x}{n^5} = \infty$ (Note: The denominator is a constant!)

G. $\lim\limits_{x\to\infty} \frac{\sqrt[3]{x}}{e^x \ln x} = 0$ (e^x beats $x^{\frac{1}{3}}$, $\ln x$ helps e^x, though only slightly)

H. $\lim\limits_{x\to\infty} \frac{\sqrt{x^7+x}}{\sqrt[3]{x^{10}-x}} = \infty$ (numerator is akin to $x^{7/2}$, denominator akin to $x^{10/3}$. $\frac{7}{2} > \frac{10}{3}$, so numerator wins)

II. A $\lim\limits_{x\to\infty} \frac{\sqrt{3x^2+4}}{x-1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim\limits_{x\to\infty} \frac{\sqrt{3+\frac{4}{x^2}}}{1-\frac{1}{x}} = \sqrt{3}$

B. $\lim\limits_{x\to\infty} \frac{e^{2x}}{e^{2x}+e^x} \frac{\frac{1}{e^{2x}}}{\frac{1}{e^{2x}}} = \lim\limits_{x\to\infty} \frac{1}{1+\frac{1}{e^x}} = 1$

C. $\lim\limits_{x\to\infty} \frac{\sqrt[3]{x}}{\ln x}$ (Form "$\frac{\infty}{\infty}$")

$\stackrel{\text{L'Hô}}{=} \lim\limits_{x\to\infty} \frac{\frac{1}{3}x^{-\frac{2}{3}}}{\frac{1}{x}} = \lim\limits_{x\to\infty} \frac{x}{3x^{2/3}} = \lim\limits_{x\to\infty} \frac{x^{\frac{1}{3}}}{3} = \infty$

D. $\lim\limits_{x\to\infty} x^2 e^{-x^2} = \lim\limits_{x\to\infty} \frac{x^2}{e^{x^2}}$ (Form "$\frac{\infty}{\infty}$")

$\stackrel{\text{L'Hô}}{=} \lim\limits_{x\to\infty} \frac{2x}{2x\, e^{x^2}} = \lim\limits_{x\to\infty} \frac{1}{e^{x^2}} = 0$

E. $\lim_{x\to\infty} x\cdot\tan\frac{1}{x}$ Form "$\infty\cdot 0$"

$= \lim_{x\to\infty} \frac{\tan(\frac{1}{x})}{\frac{1}{x}}$ Form "$\frac{0}{0}$"

$\overset{L'H\hat{o}}{=} \lim_{x\to\infty} \frac{\sec^2(\frac{1}{x})\cdot -\frac{1}{x^2}}{-\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{1}{\cos^2(\frac{1}{x})} = 1$

F. $\lim_{x\to\infty} x^{1/x}$ form "∞^0"

$L = \lim_{x\to\infty} x^{\frac{1}{x}}$

$\ln L = \lim_{x\to\infty} \ln(x^{\frac{1}{x}}) = \lim_{x\to\infty} \frac{1}{x}\ln(x) = \lim_{x\to\infty} \frac{\ln x}{x}$ Form "$\frac{\infty}{\infty}$"

$\overset{L'H\hat{o}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$

$\ln L = 0$, So $L = \lim_{x\to\infty} x^{\frac{1}{x}} = e^0 = 1$

G. $\lim_{x\to\infty} (1+\frac{1}{x})^x$ Form "1^∞"

$L = \lim_{x\to\infty} (1+\frac{1}{x})^x$

$\ln L = \lim_{x\to\infty} \ln(1+\frac{1}{x})^x = \lim_{x\to\infty} x\ln(1+\frac{1}{x}) = \lim_{x\to\infty} \frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}$ Form "$\frac{0}{0}$"

$\overset{L'H\hat{o}}{=} \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}}\cdot(-\frac{1}{x^2})}{-\frac{1}{x^2}} = \lim_{x\to\infty} \frac{1}{1+\frac{1}{x}} = 1$

$\ln L = 1$ So $L = e^1 = e$

H. $\lim_{x\to\infty} \frac{x\sin^2 x}{e^{2x}}$ $0 \le \sin^2 x \le 1$, so $0 \le \frac{x\sin^2 x}{e^x} \le \frac{x}{e^{2x}}$

$\lim_{x\to\infty} \frac{x}{e^{2x}}$ (Form "$\frac{\infty}{\infty}$")

$\overset{L'H\hat{o}}{=} \lim_{x\to\infty} \frac{1}{2e^{2x}} = 0$

And $\lim_{x\to\infty} 0 = 0$.

Since $\frac{x\sin^2 x}{e^{2x}}$ is sandwiched between 0 and $\frac{x}{e^x}$, $\lim_{x\to\infty} \frac{x\sin^2 x}{e^{2x}} = 0$ as well