HOMEWORK 5

TEMPLATE
MATH 2001

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ABSTRACT. This is the fifth homework assignment. The problems are from Hammack [Ham13, Ch. 2, §2.5, 2.6, 2.7, 2.9, 2.10]:

- **Chapter 2** Exercises: **Section 2.5**: 4, 6, 8.
- **Chapter 2** Exercises: **Section 2.6**: 4, 6.
- **Chapter 2** Exercises: **Section 2.7**: 2, 4, 8.
- **Chapter 2** Exercises: **Section 2.9**: 2, 3, 4, 5.
- **Chapter 2** Exercises: **Section 2.10**: 2, 4, 10.

SECTION 2.5

Problem 2.5.4. Write a truth table for the logical statement

$$\sim (P \vee Q) \vee (\sim P).$$

Solution to Problem 2.5.4. The truth table is the following: [1] ←1

Date: February 28, 2016.

I would like to take this opportunity to thank my class for their support.

[1] I worked on this problem with the entire class. You are encouraged to work together on homework assignments. However, for each problem you must write your own solution, you must indicate with whom you worked, and you must cite any resources you used in solving the problem.

P	Q	$\sim(P \vee Q)$	$\sim P$	$\sim(P \vee Q) \vee (\sim P)$
T	T	F	F	F
T	F	F	F	F
F	T	F	T	T
F	F	T	T	T

A comment on the problem. As an aside, note that, once one has the notion of logical equivalence (**Section 2.6**), and a few examples, one could easily have arrived at the final outcome since $\sim(P \vee Q)$ is logically equivalent to $\sim P \wedge \sim Q$, so that $\sim(P \vee Q) \vee (\sim P)$ is logically equivalent to $(\sim P \wedge \sim Q) \vee (\sim P)$, which is logically equivalent to $\sim P$.

Problem 2.5.6.

Solution to Problem 2.5.6.

Problem 2.5.8.

Solution to Problem 2.5.8.

Section 2.6

Problem 2.6.4.

Solution to Problem 2.6.4.

Problem 2.6.6.

Solution to Problem 2.6.6.

Section 2.7

Problem 2.7.2. Write the following as an English sentence. Say whether it is true or false. [2] ←2

$$\forall x \in \mathbb{R},\ \exists n \in \mathbb{N},\ x^n \geq 0.$$

Solution to Problem 2.7.2. As an english sentence, this is: *For all real numbers x, there is a natural number n such that x^n is greater than or equal to* 0.

This statement is TRUE (one can always take $n = 2$, since the square of any real number is nonnegative).

Problem 2.7.4.

Solution to Problem 2.7.4.

Problem 2.7.8.

Solution to Problem 2.7.8.

Section 2.9

Problem 2.9.2. Translate the following sentence into symbolic logic. *The number x is positive but the number y is not positive.* [3] ←3

Solution to Problem 2.9.2. I understand the problem to be tacitly implying that x and y are real numbers. In symbolic logic, this is

$$(x, y \in \mathbb{R})\ \wedge\ (x > 0) \wedge\ \sim (y > 0).$$

[2] I worked on this problem with the entire class.

[3] I worked on this problem with the entire class.

Problem 2.9.3.

Solution to Problem 2.9.3.

Problem 2.9.4.

Solution to Problem 2.9.4.

Problem 2.9.5.

Solution to Problem 2.9.5.

Section 2.10

Problem 2.10.2. Negate the following sentence.

If x is prime, then x is not a rational number.

Solution to Problem 2.10.2. A negation of this sentence is

x being prime does not imply x is not a rational number.

By this we mean:

> *There exists an integer x, such that x being prime does not imply x is not a rational number*,

and this is logically equivalent to

> *There exists an integer x, such that x is prime and x is a rational number.*

The reason is the following. The sentence

If x is prime, then x is not a rational number is meant to be understood as

For all integers x, if x is prime, then x is not a rational number.

In logical notation,

$$\forall x \in \mathbb{Z},\ p(x) \implies q(x),$$

where for each $x \in \mathbb{Z}$, the statements $p(x)$ and $q(x)$ are:

$$p(x) : x \text{ is prime.}$$

$$q(x) : x \text{ is not a rational number.}$$

The negation of the statement in the problem is then

There exists an integer x, such that x being prime does not imply x is not a rational number.

In logical notation,

$$\exists x \in \mathbb{Z},\ \sim (p(x) \implies q(x)),$$

This is of course logically equivalent to

There exists an integer x, such that x is prime and x is a rational number.

In logical notation,

$$\exists x \in \mathbb{Z},\ p(x) \wedge \sim q(x)).$$

Problem 2.10.4.

Solution to Problem 2.10.4.

Problem 2.10.10.

Solution to Problem 2.10.10.

A few examples from class

This is a test of summation and product. This is what it looks like between double dollar signs:

$$\sum_{i=1}^{n} i, \quad \prod_{i=1}^{n} i.$$

This is what it looks like between single dollar signs: $\sum_{i=1}^{n} i$, $\prod_{i=1}^{n} i$. To make this look like the formula above, write `\displaystyle`:
$\displaystyle\sum_{i=1}^{n} i$, $\displaystyle\prod_{i=1}^{n} i$.

References

[AM69] M. F. Atiyah and I. G. Macdonald, *Introduction to commutative algebra*, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1969. MR 0242802 (39 #4129)

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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