PRACTICE MIDTERM I

MATH 3140

Friday February 25, 2011.

Name

Please answer the all of the questions, and show your work.

1	2	3	4	5	
10	10	10	10	10	total

Date: February 20, 2011.

1
10 points

1 . Define a binary operation $*$ on $X = \mathbb{R} - \{1\}$ by

$$a * b := ab - a - b + 2,$$

for all $a, b \in X$. Is $(X, *)$ a group? Explain your answer.

2
10 points

2 (a). [3 points] How many subgroups are there of $\mathbb{Z}_{42}$.

2 (b). [3 points] Is [21] a generator of $\mathbb{Z}_{100}$?

2 (c). [4 points] Are the groups $\mathbb{Z}_6 \times \mathbb{Z}_{15} \times \mathbb{Z}_8$ and $\mathbb{Z}_3 \times \mathbb{Z}_{24} \times \mathbb{Z}_{10}$ isomorphic?

3
10 points

3 . Consider the dihedral group D_3; that is the group of symmetries of an equilateral triangle. Using the same notation as in class, let $R \in D_3$ correspond to clockwise rotation by $\pi/3$ radians, and let $D \in D_3$ correspond to flipping through a chosen vertex.

3 (a). Find $i \in \mathbb{Z}$ and $j \in \{0, 1\}$ such that $R^2DRDR^{-1} = R^iD^j$.

3 (b). What is the order of the element RD?

3 (c). Show that D_3 is isomorphic to S_3, the symmetric group on 3 letters.

4
10 points

4 . Let $G = \{0, 1, 2, \ldots, n\}$ and let $*$ be a binary operation on G. Assume that $(G, *)$ is a group, and $*$ satisfies

(1) $a * b \leq a + b$ for all $a, b \in G$.
(2) $a * a = 0$ for all $a \in G$

Show that $n = 2^m - 1$ for some $m \in \mathbb{N}$. [Hint: use (1) and (2) to show that 0 is the identity element. Then use (2) to show that G is abelian. Then use (2) and the theorem on finitely generated abelian groups.]

5
10 points

5 . True or false. If true, explain briefly (a sentence or two). If false, provide a counter example.

5 (a). Every cyclic group is abelian. [ ]

5 (b). Every abelian group is cyclic. [ ]

5 (c). An element g of a group G has order $n > 0$ if and only if $g^n = e$. [ ]

5 (c). A cyclic group has a unique generator. [ ]

5 (e). If H and H' are subgroups of a group G, then $H \cup H'$ is a subgroup. [ ]

5 (f). There exists a finite abelian group of every order $n \in \mathbb{N}$. [ ]