EUCLIDEAN AND NON-EUCLIDEAN GEOMETRY
MATH 3210

HOMEWORK 1

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1. Exercises

Exercise A. Let $A = \{1, 2, 4, 6\}$, $B = \{3, 2, 5\}$ and $C = \{2, 5, 10\}$. Find the following sets:

(1) $A \cup B$.

(2) $A \cap B$.

(3) $A - B$.

(4) $B - A$.

(5) $(B \cup C) - A$.

(6) $(A \cup C) \cap B$.

(7) $\mathscr{P}(B)$.

Exercise B. Let J and B be sets. For each $j \in J$, let A_j be a set. Show the following:

(1) $B \cup \left(\bigcap_{j \in J} A_j \right) = \bigcap_{j \in J} (B \cup A_j)$.

(2) $B \cap \left(\bigcup_{j \in J} A_j \right) = \bigcup_{j \in J} (B \cap A_j)$.

(3) $B - \left(\bigcap_{j \in J} A_j \right) = \bigcup_{j \in J} (B - A_j)$.

(4) $B - \left(\bigcup_{j \in J} A_j \right) = \bigcap_{j \in J} (B - A_j)$

Exercise C. Suppose that $f : A \to B$ is a map of sets, and let $C \subseteq A$.

(a) Prove or give a counter example: $f(A - C) \subseteq f(A) - f(C)$.

(b) Prove or give a counter example: $f(A) - f(C) \subseteq f(A - C)$.

(c) If f is injective, is it true that $f(A - C) = f(A) - f(C)$?

(d) If f is bijective, is it true that $f(A - C) = B - f(C)$?

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Exercise D. Define a relation on $\mathbb{N} \times \mathbb{N}$ by

$$(a, b) \sim (c, d) \iff a + d = b + c.$$

(1) Show that $\sim$ is an equivalence relation.

(2) Show that if $(a, b) \sim (a', b')$ and $(c, d) \sim (c', d')$ then $(a + c, b + d) \sim (a' + c', b' + d')$.

(3) Show that if $(a, b) \sim (a', b')$ and $(c, d) \sim (c', d')$ then $(ac + bd, bc + ad) \sim (a'c' + b'd', b'c' + a'd')$.

(4) Let $Z = (\mathbb{N} \times \mathbb{N})/\sim$. Show that there is a map

$$+ : Z \times Z \to Z$$

defined by $[(a, b)] + [(c, d)] = [(a + c, b + d)]$.

(5) Let $Z = (\mathbb{N} \times \mathbb{N})/\sim$. Show that there is a map

$$\cdot : Z \times Z \to Z$$

defined by $[(a, b)] \cdot [(c, d)] = [(ac + bd, bc + ad)]$.

(6) Let $0_Z := [(1, 1)]$. Show that for all $z \in Z$, $0_Z + z = z$.

(7) For all $z \in Z$, show that there exists $z' \in Z$ such that $z' + z = 0_Z$.

(8) For all $x, y, z \in Z$, show that $(x + y) + z = x + (y + z)$.

(9) For all $x, y \in Z$, show that $x + y = y + x$.

(10) Let $1_Z = [(1, 0)]$. Show that for all $z \in Z$, $1_Z \cdot z = z$.

(11) For all $x, y \in Z$, show that $x \cdot y = y \cdot x$.

(12) For all $x, y, z \in Z$, show that $x \cdot (y + z) = x \cdot y + x \cdot z$.