SAMPLE MIDTERM II
ANALYSIS 1

MATH 3100

Friday November 8, 2013

Name | ______________________________

Please answer all of the questions, and show your work.
All solutions must be explained clearly to receive credit.

1	2	3	4	5	
10	10	10	10	10	Total

Date: November 4, 2013.

1
10 points

1. Show that the image of a compact set under a continuous map is compact; i.e. if $S \subseteq \mathbb{R}$, $f : S \to \mathbb{R}$ is a continuous map, and $K \subseteq S$ is compact, then $f(K) \subseteq \mathbb{R}$ is compact.

EXTRA CREDIT: Prove this for arbitrary topological spaces. In other words, show that if $f : X \to Y$ is a continuous map of topological spaces and $K \subseteq X$ is compact, then $f(K)$ is compact.

2
10 points

2. Define a sequence $\{s_n\}_{n\in\mathbb{N}}$ recursively by $s_1 = \sqrt{12}$ and $s_{n+1} = \sqrt{12 + s_n}$ for all $n \in \mathbb{N}$. Show that this sequence converges and find the limit. [Hint: Show the sequence is monotone and bounded.]

3
10 points

3.(a). Let $\{s_n\}_{n\in\mathbb{N}}$ be a bounded sequence. Define the limit inferior, $\liminf\{s_n\}$.

3.(b). Let $s_n = 3\sin^2(n\pi/2)$. Find the limit superior and limit inferior of the sequence $\{s_n\}_{n\in\mathbb{N}}$ (you must show your work, but you do not need to give a detailed proof).

4
10 points

4. Show that if f is continuous, then $|f|$ is continuous; i.e. if $S \subseteq \mathbb{R}$ and $f : S \to \mathbb{R}$ is a continuous map, then $|f| : S \to \mathbb{R}$ is also a continuous map. Recall that by definition, for each $x \in S$, $|f|(x) := |f(x)|$.

5
10 points

5. True or False. *You do NOT need to justify your answer.*

5.(a) A union of compact sets is compact.

5.(b) If $\{s_n\}_{n\in\mathbb{N}}$ is a sequence of positive numbers and $\lim s_{n+1}/s_n = L < 1$, then the sequence $\{s_n\}_{n\in\mathbb{N}}$ converges to 0.

5.(c) If $\{s_n\}_{n\in\mathbb{N}}$ is a sequence of positive numbers and if $\lim s_{n+1}/s_n = 1$, then the sequence $\{s_n\}_{n\in\mathbb{N}}$ does not converge.

5.(d) A monotone bounded sequence is convergent.

5.(e) Every bounded monotone sequence is Cauchy.

5.(f) The image of a closed set under a continuous map is closed.

5.(g) Let C be a non-empty subset of $\mathbb{R}$. C is compact if and only if every sequence in C has a subsequence that converges to a point in C.

5.(h) Let $\{s_n\}$ and $\{t_n\}$ be bounded sequences. Then

$$\limsup\{s_n + t_n\} = \limsup\{s_n\} + \limsup\{t_n\}.$$

5.(i) Let $S \subseteq \mathbb{R}$, let $f : S \to \mathbb{R}$ and let $c \in \mathbb{R}$ be a limit point of S. Then $\lim_{x\to c} f(x) = L$ if and only if for every sequence $\{s_n\}$ in S that converges to c, with $s_n \neq c$ for all n, the sequence $\{f(s_n)\}$ converges to L.

5.(j) If f and g are continuous functions (with common domain), then $f + g$ is a continuous function.