The purpose of this review sheet is to better your understanding of series. I suggest writing on a seperate sheet of paper. For many of the terms, you may need to use the index in the textbook.

1. For each of the terms below, write the book's definition and the definition in your own words and any formulas or facts associated with it.

   (a) Sequence
   (b) Series
   (c) Geometric Series
   (d) Alternating Series
   (e) Power Series
   (f) Taylor Series
   (g) General term
   (h) Index
   (i) $P_n(x)$
   (j) $E_n(x)$
   (k) Radius of convergence
   (l) Interval of convergence

2. If you have a sequence, a_n, how can you form a series out of it?

3. If you have a series $S = \sum_{k=1}^{\infty} b_k$, what are two different sequences you can get from it?

4. Give the book's definition and explain in your own words what it means for a sequence to converge?

5. Give the book's definition and explain in your own words what it means for a series to converge?

6. If the series $\sum_{n=1}^{\infty} a_n$ converges, what must be true of $\lim_{n\to\infty} a_n$?

7. State all of the convergence tests for series (there are 6 of them). For each test,

   (a) Find examples of series that converges by the test.
   (b) Find examples of series that diverges by the test.
   (c) Each test has conditions that a series must meet in order to use that test. For each condition of each test, find a series that does not fulfill that condition so that the test gives an incorrect result. (For example: you could find an alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ for which $\lim_{n\to\infty} a_n \neq 0$)

8. What does it mean for a series to be absolutely convergent?

9. What does it mean for a series to be conditionally convergent?

10. If it is possible, give an example of a series which is absolutely convergent but not conditionally convergent.

11. If it is possible, give an example of a series which is conditionally convergent but not absolutely convergent.

12. What is the formula for the sum of a geometric series?

13. Let $f(x) = x^6 + 4x^4 - x^3 + x - 5$. Find the following:

    (a) $P_4(x)$ centered at $x = 0$.

    (b) $P_4(x)$ centered at $x = 1$.

    (c) $P_5(x)$ centered at $x = 0$.

    (d) $P = \lim_{n\to\infty} P_n(x)$ centered at $x = 0$.

14. Write the formula for the Taylor series of a function $f(x)$.

15. For each of the following functions, write the Taylor series centered at the given value of x, determing the interval of convergence, and for a value of x in the interval of convergence, find a bound on $E_n(x)$.

    (a) e^x about $x = 0$.

    (b) $\sin x$ about $x = 0$.

    (c) $\cos x$ about $x = 0$.

    (d) $ln(x)$ about $x = 1$.

    (e) $ln(x + 1)$ about $x = 0$.

    (f) $(1 + x)^p$ about $x = 0$.

    (g) $\frac{1}{1+x}$ about $x = 0$.

16. For each of the functions $f(x)$ given above, find the taylor series for $x^2 f(3x)$

17. The text (Hughes-Hallet Calculus 5th Edition) a review section for series on pages 497-500

    (a) For power series practice, do problems 49, 53, 54, 55, 56, 57

    (b) For absolute convergence practice, do problems 18-21

    (c) For general convergence practice, do some of problems 28-47 on page 498 of the text. Remember that part of being good at convergence problems is being able to quickly recognize which test is appropriate.