MATH 2300: CALCULUS 2

April 6, 2011

TEST 3:

1. Does the following series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

You must justify your answer to receive credit.

2. Suppose $a_n \geq 0$ and $b_n \geq 0$ for all n. Also suppose that the series $\sum a_n$ converges, and that $\lim_{n\to\infty} \frac{a_n}{b_n} = 2$. Which of the following are true, and which are false? Provide a brief explanation for each of your answers.

(a) The sequence a_n converges.

(b) $\lim_{n\to\infty} a_n = 1$.

(c) The series $\sum b_n$ is convergent.

(d) The sequence b_n converges.

(e) The series $\sum \frac{b_n}{a_n}$ is convergent.

3. Suppose that $\frac{dy}{dx} = f(x)$, where $f(x)$ is shown in the graph below.

quartic.pdf

Which one of the slope fields (i), (ii), (iii), (iv) below could be the slope field of this differential equation? Explain briefly.

OtherProblemA.pdf

OtherProblemB.pdf

(i) (ii)

`OtherProblemC.pdf` `OtherProblemD.pdf`

(iii) (iv)

4. (a) Write down the second degree Taylor polynomial $P_2(x)$ approximating

$$f(x) = e^{\sin(x)}$$

near $x = 0$.

(b) Use your result from part (a) to approximate $e^{\sin(0.1)}$.

5. Suppose $f(x)$ is a function such that, for any positive integer n,

$$|f^{(n+1)}(x)| \leq n!$$

for all x.

(a) Use the Lagrange Error Bound Formula to show that the error term $E_n(x) = f(x) - P_n(x)$, where $P_n(x)$ is the Taylor polynomial of degree n for $f(x)$ near $x = 0$, satisfies

$$|E_n(x)| \leq \frac{|x|^{n+1}}{n+1}.$$

(b) Show that, for f as above, the Taylor series at $x = 0$ for $f(x)$ converges to $f(x)$ for $-1 \leq x \leq 1$.

6. What's the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}?$$

Please show all of your work (and don't forget to check the endpoints).

7. Suppose the series

$$\sum_{n=5}^{\infty} C_n (x-3)^n$$

converges when $x = 0$ but diverges when $x = 9$. For each of the following values of x, determine whether the series converges or diverges there, or if there's not enough information to say. Explain each of your answers briefly.

(a) $x = 1$

(b) $x = -5$

(c) $x = -2$

8. (a) Solve the initial value problem

$$\frac{dy}{dx} = \frac{\cos(x)}{e^y}, \quad y(0) = 0.$$

Please make sure you solve for y; that is, express your solution in the form

$$y = \text{ a function of } x.$$

(b) Show that, if y is the function you found in part (a) above, then

$$y'' = -e^{-y}.$$

In computing y'', you may want to use the fact that $\cos^2 x + \sin^2 x = 1$ for any x.

9. Which one of the functions (a), (b), (c), (d) sketched below could possibly have Taylor series

$$3 + 5(x-2) - 2(x-2)^2 + \cdots ?$$

Please explain.