MATH 2300: CALCULUS 2

March 2, 2011

TEST 2

I have neither given nor received aid on this exam.

Name:____

- ◯ **001** A. Pajer . . . . (8am)
- ◯ **002** B. Katz-Moses . . . . (9am)
- ◯ **003** W. Stanton . . . . (10am)
- ◯ **004** J. Wiscons . . . .(11am)
- ◯ **005** A. Lizzi . . . . (12pm)
- ◯ **006** E. Stade . . . . (1pm)
- ◯ **007** C. Scherer . . . . (1pm)
- ◯ **008** M. Roy . . . . (2pm)

If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show all of your work, and give adequate explanations.

You are **allowed and encouraged** to use

- your graphing calculator
- the table of formulas attached to the exam

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	21 pts	
2	10 pts	
3	10 pts	
4	7 pts	
5	10 pts	
6	10 pts	
7	6 pts	
8	10 pts	
9	10 pts	
10	6 pts	
TOTAL	100 pts	

1. (3pt each) For each of the following circle sequence or series. Also, circle converges or diverges. Give a calculation or brief explanation of how you know whether or not it converges.

(a) $1, 1/3, 1/9, 1/27, \ldots$

- The above is a **sequence** / **series** (circle one).
- The above **converges** / **diverges** (circle one).
- Brief explanation:

(b) $s_n = 1/n^2 + \cos(\pi n)$

- The above is a **sequence** / **series** (circle one).
- The above **converges** / **diverges** (circle one).
- Brief explanation:

(c) $\displaystyle\sum_{n=0}^{\infty} (0.9)^n$

- The above is a **sequence** / **series** (circle one).
- The above **converges** / **diverges** (circle one).
- Brief explanation:

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$.

- The above is a **sequence** / **series** (circle one).
- The above **converges** / **diverges** (circle one).
- Brief explanation:

(e) $1 + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \dfrac{1}{4^4} + \cdots$.

- The above is a **sequence** / **series** (circle one).
- The above **converges** / **diverges** (circle one).
- Brief explanation:

(f) $\displaystyle\sum_{n=10}^{\infty} \frac{1}{n^{-1/2}}$

- The above is a **sequence** / **series** (circle one).
- The above **converges** / **diverges** (circle one).
- Brief explanation:

(g) $s_n = \dfrac{3}{2} s_{n-1};\quad s_1 = 5$

- The above is a **sequence** / **series** (circle one).
- The above **converges** / **diverges** (circle one).
- Brief explanation:

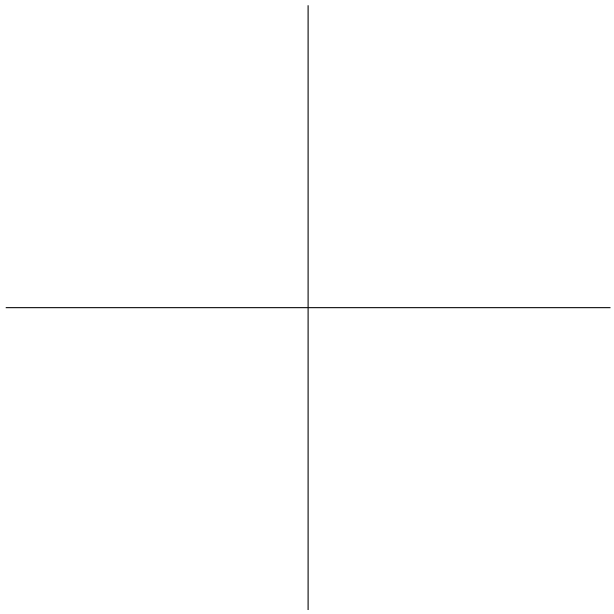
2. (a) (2pt) Draw a sketch representing the area enclosed by the curves $x^2 + y^2 = 1$ and $y = -x + 1$, with $x \geq 0$, $y \geq 0$.

(b) (4pt) Using the sketch, draw a vertical slice of the region with width dx, and find the area of that slice.

(c) (4pt) Using the area of the slice you found in part (b), write down and compute an integral that gives area of the shaded region.

3. (10pt) Consider the functions $r = 4\sin(2\theta)$ and $r = 2\sin(2\theta)$ for θ between 0 and $\pi/2$.

(a) (4pt) Using the axes below, draw a sketch of the two functions, and shade the polar region $2\sin(2\theta) < r < 4\sin(2\theta)$, $0 \le \theta \le \pi/2$. Feel free to use your calculator.



(b) (6pt) Compute the area of the shaded region.

4. (7pt) An ant walked along the graph of the curve $y = x^{3/2}$ between the points $(1, 1)$ and $(4, 8)$. What was the total distance he traveled?

5. (10pt) Jesse Genius is the world's greatest calculus student. The first calculus problem took one hour for Jesse to solve. The second problem only took Jesse $\frac{1}{2}$ of an hour to solve. The third problem took $\frac{1}{4}$ of an hour, and the fourth problem took $\frac{1}{8}$ of an hour. Jesse has no problem maintaining this pattern.

(a) (4pt) How long did it take Jesse to do just the n^{th} problem?

(b) (3pt) How much total time did Jesse spend on the first 1000 problems? Give an exact answer (no calculator approximations). Do not use summation notation or "...".

(c) (3pt) How long will it take Jesse Genius to solve infinitely many calculus problems? Give an exact answer (no calculator approximations). Do not use summation notation or "...".

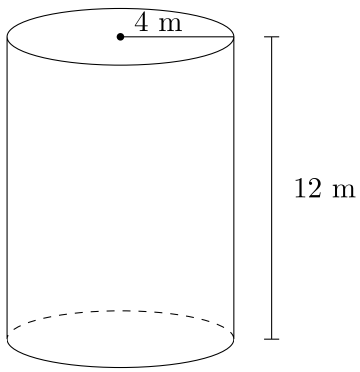
6. (10pt) The region bounded by $y = f(x)$, $y = 0$, $x = 0$, and $x = 5$ is rotated around the x-axis. Estimate the volume of the resulting solid by using the information in the table below. Assume f is positive.

x	0	1	2	3	4	5
$f(x)$	0.9	1.1	1.2	1.0	0.8	0.9

7. (6pt) Your friend says, "Since $\displaystyle\lim_{n\to\infty} \frac{1}{n^3} = 0$, we know that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges." Your friend is right that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges, but your friend's **reasoning** is wrong. Give an example where your friend's reasoning will lead him to the wrong conclusion. That is, give an example of a series $\sum a_n$ such that $\lim_{n\to\infty} a_n = 0$, but $\sum a_n$ diverges.

8. (10pt) Using the Comparison Test, show whether $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x^3+1}}$ converges or diverges.

9. (10pt) A compressible liquid has density which varies with height. At the level of h feet above the bottom, the density is $36(16-h)\dfrac{kg}{m^3}$. How many kilograms will the container below hold?



10. (6pt) Do **exactly one** of the following three problems. Clearly indicate which problem you are attempting.

(a) A bowl is shaped like a hemisphere with diameter 30cm. A ball with diameter 10cm is placed in the bowl and water is poured into the bowl to a depth of 4cm. Find the volume of water in the bowl. Assume the ball does not float.

(b) Let f be a function such that $0 \le f(x) \le 1$ and $f'(x) > 0$ for all x. Also, let s_n be an increasing sequence. Does the sequence $f(s_n)$ converge or diverge? Briefly explain your reasoning.

(c) Using the procedures we discussed in class, a student found the volume of a solid to be given by the integral $\int_0^7 \pi(49 - h^2)dh$ where h measures length. At what solid was the student looking?