MATH 2300: CALCULUS 2

February 2, 2011

TEST 1

On my honor as a University of Colorado at Boulder student I have neither given nor received unauthorized assistance on this exam.

Name:____

- ◯ **001** A. Pajer .... (8am)
- ◯ **002** B. Katz-Moses .... (9am)
- ◯ **003** W. Stanton .... (10am)
- ◯ **004** J. Wiscons .... (11am)
- ◯ **005** A. Lizzi .... (12pm)
- ◯ **006** E. Stade .... (1pm)
- ◯ **007** C. Scherer .... (1pm)
- ◯ **008** M. Roy .... (2pm)

In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show all of your work, and give adequate explanations.

You are **allowed and encouraged** to use

- your graphing calculator, and
- the table of integrals from our textbook that has been included at the end of the exam.

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	12 pts	
2	6 pts	
3	14 pts	
4	18 pts	
5	6 pts	
6	16 pts	
7	12 pts	
8	10 pts	
9	6 pts	
TOTAL	100 pts	

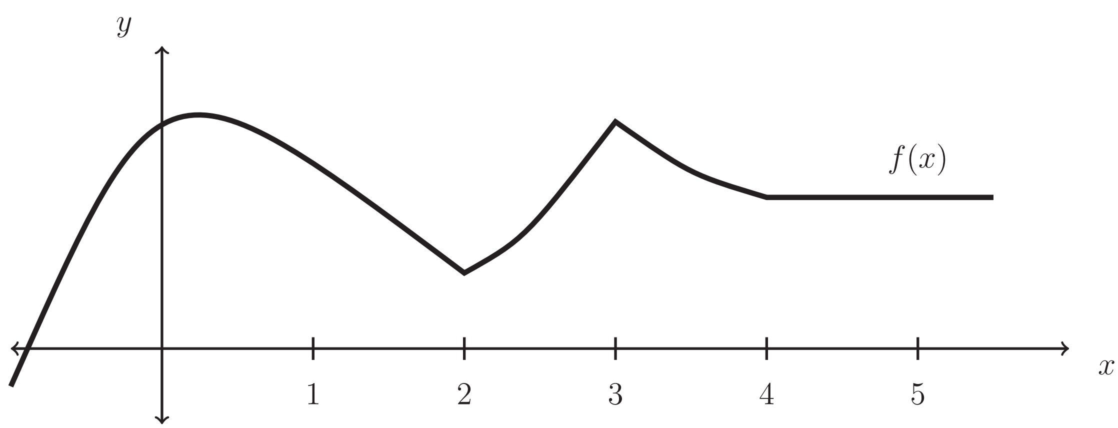
1. (12 pt.) Let $f(x) = e^{x^2}$.

(a) Approximate $\int_0^4 f(x)\,dx$ using the Midpoint Rule with 2 equal subdivisions. **Show your work.**

(b) Does this approximation give an underestimate or overestimate of the actual value? **Explain your reasoning.**

2. (6 pt.) The graph of a function $f(x)$ is given below. **Carefully explain** why the following statement is true.

"If the Trapezoid Rule is used to estimate the area under f from 2 to 4, then it is much better to use 2 equal subdivisions than to use 3 equal subdivisions."



3. (14 pt.) Each of the following integrals can be evaluated by first making a substitution and then directly applying an integration formula from the attached table. **Fill in the blanks**. Please **DO NOT EVALUATE** these integrals.

(a) To evaluate $\displaystyle\int \frac{1}{x\sin(\ln(x))}\,dx$,

- first make the substitution $u =$__________________;
- then use integration formula #_________.

(b) To evaluate $\displaystyle\int \frac{(2\sin x + 5)\cos x}{\sin^2 x + 7}\,dx$,

- first make the substitution $u =$__________________;
- then use integration formula #_________.

4. (18 pt.) Choose an appropriate method for evaluating each of the integrals below by checking **only one** of the boxes, and filling in the blanks. Some of them may have more than one possible approach, but choose only **one**. Please **DO NOT EVALUATE** the integrals, but feel free to check your answers using the extra space. (For integrals that you would do using partial fractions, you DO NOT have to solve for the unknown coefficients. Just show how you would SET THINGS UP, by rewriting the rational function in question as a sum of terms.)

(a) An appropriate method for evaluating $\displaystyle\int x \sin x \, dx$ is to

☐ use integration by parts with $u =$__________ and $v' =$__________ .

☐ use partial fractions by first rewriting $x \sin x$ as ____________________ .

☐ use a trigonometric substitution with $x =$____________________ .

(b) An appropriate method for evaluating $\displaystyle\int \frac{x^3}{\sqrt{4-x^2}} \, dx$ is to

☐ use integration by parts with $u =$__________ and $v' =$__________ .

☐ use partial fractions by first rewriting $\dfrac{x^3}{\sqrt{4-x^2}}$ as ____________________ .

☐ use a trigonometric substitution with $x =$____________________ .

(c) An appropriate method for evaluating $\displaystyle\int \frac{2x^2-10}{x(x^2-1)} \, dx$ is to

☐ use integration by parts with $u =$__________ and $v' =$__________ .

☐ use partial fractions by first rewriting $\dfrac{2x^2-10}{x(x^2-1)}$ as ____________________ .

☐ use a trigonometric substitution with $x =$____________________ .

5. (6 pt.) Use the trig substitution $x = \tan\theta$ to show that

$$\int \frac{dx}{1+x^2} = \arctan(x) + C.$$

(Please use the indicated method; do not use integral tables or other methods). You may want to recall that $d(\tan\theta)/d\theta = \sec^2\theta$ and that $1 + \tan^2\theta = \sec^2\theta$.

6. (16 pt.) Determine if each of the following integrals converges or diverges. If the integral converges, find its value. If it diverges, write "DIVERGES", and explain why. **You must show all work.** Calculator approximations are not acceptable.

(a) $\displaystyle\int_0^\infty e^{-x}\,dx$

(b) $\displaystyle\int_0^1 -\ln x\,dx$

7. (12 pt.) By following the steps below, establish the integration formula:

$$\int \frac{1}{x(x^2+a)}\,dx = \frac{1}{a}\ln|x| - \frac{1}{2a}\ln|x^2+a| + C, \qquad \text{for } a > 0.$$

(a) Use the method of partial fractions to break up $\dfrac{1}{x(x^2+a)}$ into a sum of two fractions. Note that $a > 0$, so $x^2 + a$ can not be factored.

(b) Use your answer from the previous part to evaluate

$$\int \frac{1}{x(x^2+a)}\,dx.$$

8. (10 pt.) The rate of oil production in a new well is given by

$$R(t) = e^{-t}\sin(2\pi t) + e^{-t}$$

where t is the time in years.

(a) Find an antiderivative for $R(t)$. The integration table should be helpful.

(b) Determine how much oil is produced by the well over its entire lifetime (from $t = 0$ to $t = \infty$).

9. (6 pt.) Assume that you are given a function $f(x)$ such that

$$\begin{aligned} &f(0) = f'(0) = f''(0) = 1, \text{ and} \\ &f(1) = f'(1) = f''(1) = e. \end{aligned}$$

Determine the exact value of

$$\int_0^1 x f''(x)\, dx.$$