MATH 2300: CALCULUS 2

May 2, 2011

FINAL EXAM

I have neither given nor received aid on this exam.

Name:_____

◯ **001** A. Pajer ..... (8am)
◯ **002** B. Katz-Moses ..... (9am)
◯ **003** W. Stanton ..... (10am)
◯ **004** J. Wiscons ..... (11am)
◯ **005** A. Lizzi ..... (12pm)
◯ **006** E. Stade ..... (1pm)
◯ **007** C. Scherer ..... (1pm)
◯ **008** M. Roy ..... (2pm)

If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show all of your work, and give adequate explanations.

You are **allowed and encouraged** to use your calculator, except where indicated.

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	12 pts	
2	15 pts	
3	12 pts	
4	9 pts	
5	6 pts	
6	10 pts	
7	10 pts	
8	10 pts	
9	6 pts	
10	10 pts	
TOTAL	100 pts	

1. Evaluate the integral

$$\int (x^5 + x^4)(5x^4 + 4x^3)\, dx$$

in three different ways:

(a) By multiplying out the integrand, and then integrating term by term.

(b) By substitution: put $u = x^5 + x^4$.

(c) By parts: put $u = x^5 + x^4$ and $dv = (5x^4 + 4x^3)\,dx$. [The integral you end up with on the right hand side should look remarkably similar to the one you started with; you should be able to do some algebra to solve for this integral.]

2. For each part of this problem, state which integration technique you would use to evaluate the integral, but **DO NOT** evaluate the integral. If your answer is **substitution**, also list u and du; if your answer is **integration by parts**, also list u, dv, du and v; if your answer is **partial fractions**, set up the partial fraction decomposition, but do not solve for the numerators; if your answer is **trigonometric substitution**, write which substitution you would use.

(a) $\displaystyle\int \cos 5x \sin^2 5x \, dx$

(b) $\displaystyle\int \frac{x^3}{\sqrt{9-x^2}} dx$

(c) $\displaystyle\int \frac{dx}{(x^2+1)(x-3)^2}$

(d) $\int \sqrt{x} \ln x \, dx$

(e) $\int \frac{3e^x}{5 - e^x} dx$

3. Let

$$f(x, y) = xe^x \cos y - ye^x \sin y.$$

(a) Find $f_x(x, y)$.

(b) Find $f_y(x, y)$.

(c) Show that

$$\frac{\partial}{\partial x} f_x(x, y) + \frac{\partial}{\partial y} f_y(x, y) = 0.$$

4. (a) Find the equation of the sphere that's centered at $(6, 4, 2)$, and passes through the point $(3, 5, 7)$.

(b) Circle the correct answer: the intersection of the above sphere with the plane $x = 0$ is

(i) a point;

(ii) a circle;

(iii) empty; that is, the plane and the sphere do not intersect.

If the answer is (i), give the coordinates of the point; if it's (ii), give the center and radius of the circle; if it's (iii), explain why.

(c) Circle the correct answer: the intersection of the above sphere with the plane $z = 0$ is

(i) a point;

(ii) a circle;

(iii) empty; that is, the plane and the sphere do not intersect.

If the answer is (i), give the coordinates of the point; if it's (ii), give the center and radius of the circle; if it's (iii), explain why.

5. Consider the series

$$\sum_{n=1}^{\infty} a_n.$$

If

$$\lim_{n\to\infty} a_n = 0,$$

must the series converge? If the answer is "Yes," please explain why. If the answer is "No," give a counterexample – that is, give an example of a series

$$\sum_{n=1}^{\infty} a_n$$

such that

$$\lim_{n\to\infty} a_n = 0,$$

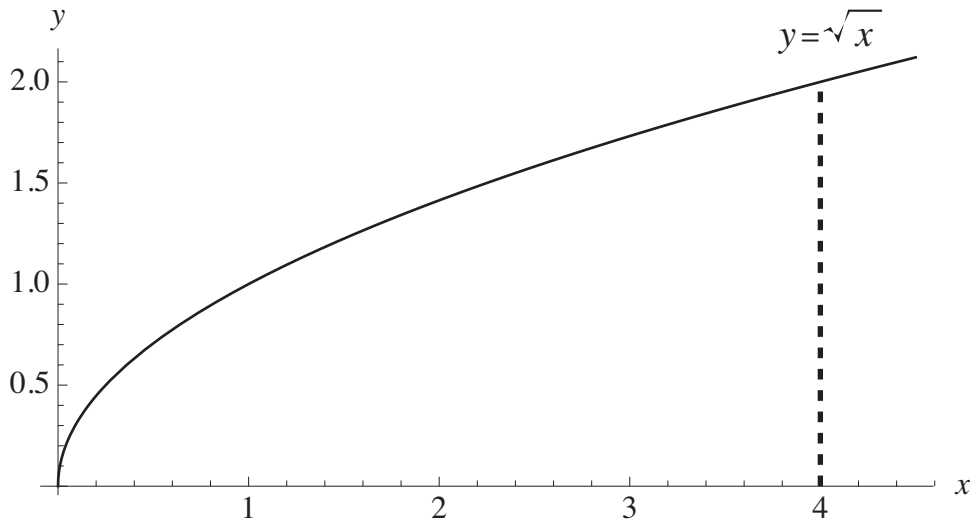
but the series diverges.

6. Find

$$\int_0^1 \int_0^x e^{x^2} \, dy \, dx.$$

Please show all work: if you need to use any techniques of integration to do either the integral in x or the integral in y, then specify how you're doing this, as in problem (2) above; also, carry these techniques through to the final answer (that is, don't just plug the integral into your calculator).

7. Consider the region R in the xy plane bounded by the curve $y = \sqrt{x}$, the x axis, and the vertical line $x = 4$.



(a) Write down two *different* iterated integrals – one in which you first integrate in x, and then in y; the other in which you first integrate in y, and then in x – that represent the mass of a plate situated on the above region R, whose density at any point (x, y) on that plate is given by $\delta(x, y)$.

(b) Find the mass of the plate described in part (a) of this problem, if

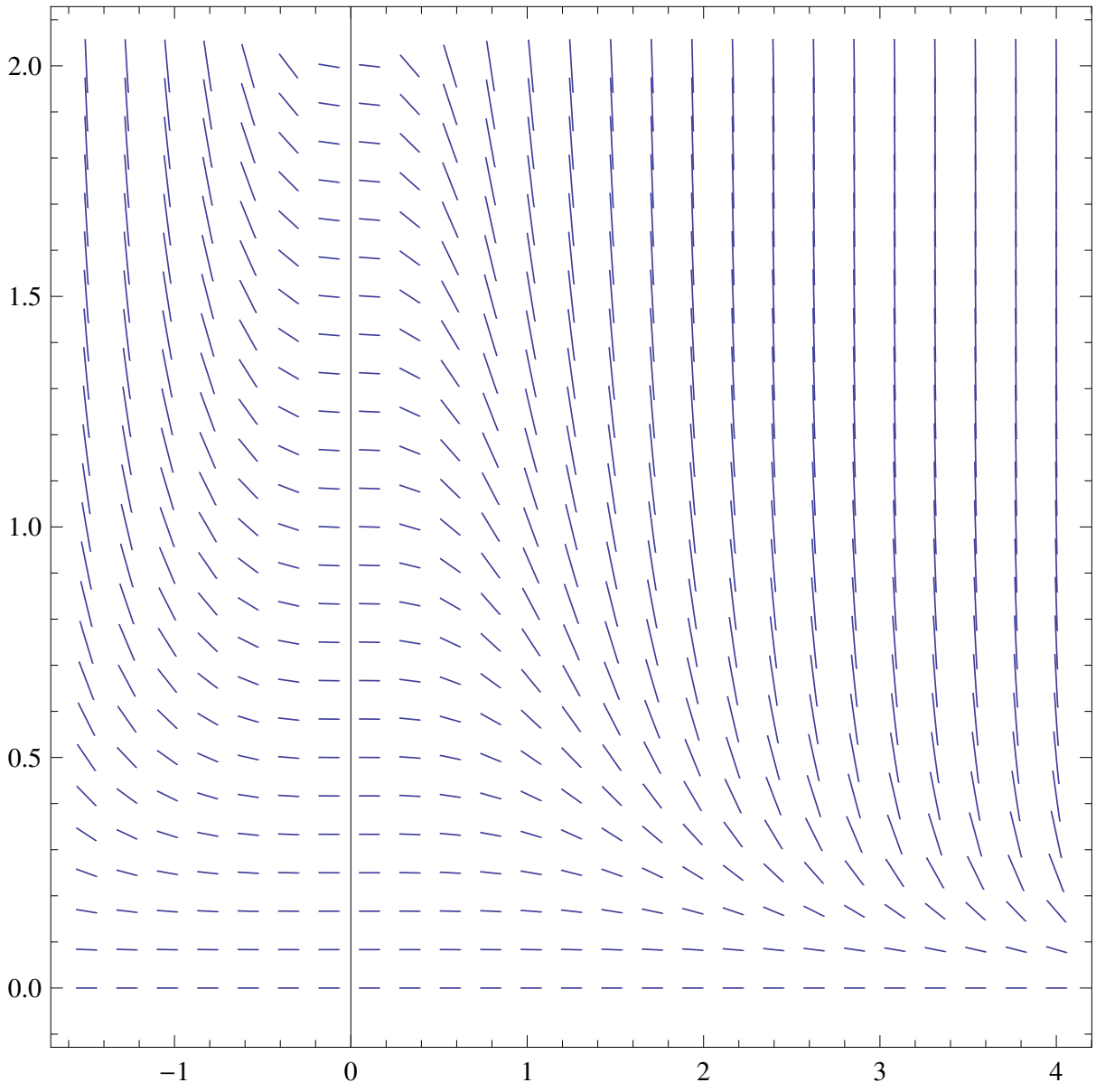
$$\delta(x, y) = \frac{\sin x}{\sqrt{x}}.$$

Please show your work (solve any integrals by hand, without using your calculator).

8. (a) Solve the initial value problem

$$\frac{dy}{dx} = -x^2y^2, \quad y(1) = 1.$$

(b) Sketch the solution you found in part (a) on the slope field below.



9. The power series $\sum_{n=1}^{\infty} C_n x^n$ diverges at $x = 4$ and converges at $x = 2$. At $x = -5$, the series

(a) Converges.

(b) Diverges.

(c) Cannot be determined.

Please explain your answer.

10. (a) Find the second degree Taylor polynomial $P_2(x)$ about $x = 0$ for

$$f(x) = \frac{1}{e^x + e^{-x}}.$$

(b) Use your answer to part (a) above to approximate $\dfrac{1}{e + e^{-1}}$.

(c) Given that $f'''(x)$ is positive for $0 \leq x \leq 1$, do you think your estimate from part (b) is an overestimate or an underestimate of $\dfrac{1}{e + e^{-1}}$? Please explain. (You can check your answer by plugging $\dfrac{1}{e + e^{-1}}$ into your calculator, but please also explain how you could have predicted this result *without* a calculator.)