HOMEWORK 1

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1. Exercises 0

Exercise 1 (# 0.12). Fill in the exercise here. Some more examples are below. This is problem number 12 in §0 of Fraleigh [2]. Other books to read are Gallian [3], and Bourbaki [1].

2. Exercises 1

3. Exercises 4

Exercise 2 (# 4.34). Let G be a group with a finite number of elements. For any $g \in G$, we are to show that there exists a number $n_g \in \mathbb{N}$ depending on g, such that $g^{n_g} = e$, where $e \in G$ is the identity element.

We can begin to list the elements of G obtained by taking powers of g. I.e.

$$e, g, g^2, g^3, \ldots$$

Since there are a finite number of elements in the group, this list must end at some point, and so we must have that there are numbers $n \neq m \in \mathbb{Z}$ with $0 \leq m \leq |G|$ and $0 \leq n \leq |G|$ such that $g^m = g^n$. In other words, $g^{n-m} = e$. So we may take $n_g = n - m$.

Exercise 3 (# 4.41). Let G be a group, and let $g \in G$. The problem is to show that the map $i_g : G \to G$ given by $i_g(x) = gxg^{-1}$ is an isomorphism.

The first thing to check is that i_g is a homomorphism; in other words, we must check that $i_g(xy) = i_g(x)i_g(y)$. To do this, consider that

$$i_g(x)i_g(y) := gxg^{-1}gyg^{-1} = gxyg^{-1} =: i_g(xy).$$

We must now show that i_g is bijective. To begin, let us show it is one-to-one. So let $x, y \in G$. We will show that $i_g(x) = i_g(y)$ only if $x = y$. Indeed, if

$$i_g(x) := gxg^{-1} = i_g(y) := gyg^{-1}$$

then composing on the left with g^{-1} and on the right with g gives that $x = y$.

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Let us now show that i_g is onto. In other words, given $y \in G$, we must find an $x \in G$ such that $i_g(x) = y$. By the definition of i_g, this means that $gxg^{-1} = y$. In other words, given y, if we set $x = g^{-1}yg$, then $i_g(x) = y$.

4. Some useful diagrams, formulae, etc, for other homework problems

$$\int_0^{2\pi} (\sin x) dx = 0$$

$$\mathbf{X} = \begin{pmatrix} x_1 & x_2 & \dots \\ x_3 & x_4 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Theorem A. *The theorem*

5. The first section

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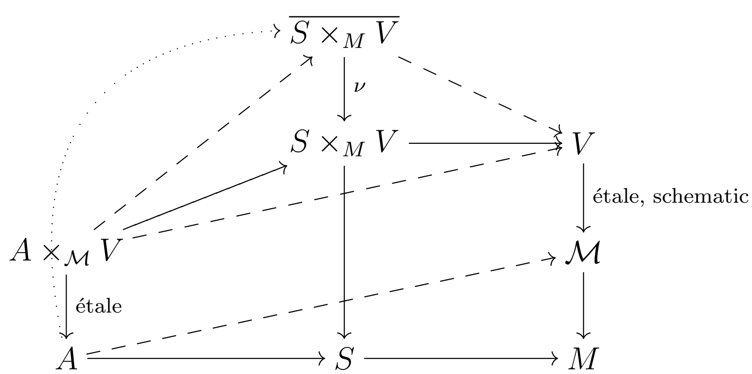
[4]

(5.1)



$$\begin{array}{ccc} \mathscr{A} & \xrightarrow{f} & B \\ \Vert & & \\ C & \longrightarrow & D \end{array}$$

References

b 1. N. Bourbaki, *Algebra I. Chapters 1–3*, Elements of Mathematics (Berlin), Springer-Verlag, Berlin, 1998, Translated from the French, Reprint of the 1989 English translation [MR0979982 (90d:00002)].

f 2. J.B. Fraleigh, *A First Course in Abstract Algebra, Seventh Edition*, Addison Wesley, Boston, 2003.

g 3. J.A. Gallian, *Contemporary Abstract Algebra, Third Edition*, D.C. Heath, Toronto, 1994.

groth 4. Alexander Grothendieck, *Cohomologie locale des faisceaux cohérents et théorèmes de Lefschetz locaux et globaux (SGA 2)*, Documents Mathématiques (Paris) [Mathematical Documents (Paris)], 4, Société Mathématique de France, Paris, 2005, Séminaire de Géométrie Algébrique du Bois Marie, 1962, Augmenté d'un exposé de Michèle Raynaud. [With an exposé by Michèle Raynaud], With a preface and edited by Yves Laszlo, Revised reprint of the 1968 French original. MR MR2171939 (2006f:14004)

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