SAMPLE MIDTERM I

MATH 4140

DATE

Name |______________________________

Please answer the all of the questions, and show your work.

Date: February 22, 2010.

1
10 points

1 . Let R be a ring (commutative with unity $1 \neq 0$) and let A be any subset of R. Show that the subset

$$\{r \in R : ra = 0 \text{ for all } a \in A\}$$

is an ideal.

2
10 points

Consider the number $\alpha := \sqrt{2 - \sqrt[3]{5}} \in \mathbb{R}$.

2 (a). Show that α is algebraic over $\mathbb{Q}$ by finding a polynomial $p(x) \in \mathbb{Q}[x]$ such that $p(\alpha) = 0$.

2 (b). Find the degree $[\mathbb{Q}(\alpha) : \mathbb{Q}]$.

3
10 points

Show that the field $\mathbb{Q}(\sqrt{2}, \sqrt[3]{2}, \sqrt[4]{2}, \ldots)$ is algebraic over $\mathbb{Q}$, but not finite.

4
10 points

4 . Suppose that $p(x) \in F[x]$ is an irreducible polynomial and E is a finite extension field of F. If $\deg p(x)$ and $[E : F]$ are relatively prime, show that $p(x)$ is irreducible over E.

5
10 points

5 . Let E be an extension field of a field F. Let $\alpha \in E$ be an element with $\alpha \notin F$. Show that multiplication by α induces a linear automorphism of E as a vector space over F. I.e.

$$\phi : E \to E$$

by

$$x \mapsto \alpha x.$$

Show that this is *not* an automorphism of E as a field.

6
10 points

Show that $x^{p^n} - x$ is the product of all monic irreducible polynomials in $\mathbb{Z}_p[x]$ of a degree d dividing n.