Perspectives on Complex Algebraic Geometry

Abstracts

Donu Arapura (Purdue)

Fibered fundamental groups and Mumford--Tate groups

The first result, which is bit technical to state, is a strong topological restriction on "large" fundamental groups of smooth complex projective varieties. The second is a calculation of the Mumford--Tate group of an etale cover of a generic curve. These seeming unrelated results were both motivated by the same question: Given a projective map of complex varieties, what sorts of restrictions are there for the (outer) action of the fundamental group of the base on the fundamental group of the general fibre or on the kernel?

Arnaud Beauville (Universite de Nice)

The stable Luroth problem

The classical Luroth problem asks whether a variety which is unirational (= dominated by projective space) is necessarily rational (= birational to projective space). We have known since 1971 that the answer is quite negative: at least in dimension 3, there are by now many examples of unirational varieties which are not rational. Hence the idea of looking for an intermediate property: a variety X is stably rational if the product of X with projective n-space is rational for n>>0. Till last year the known examples of unirational, non stably rational varieties were rare and very particular, but a new idea of Claire Voisin has significantly improved the situation. I'll explain this idea, and how it leads to new examples, particularly in dimension >3.

Fabrizio Catanese (Universitat Bayreuth)

Fujita's question on VHS over curves and a remarkable series of algebraic surfaces

I shall describe joint work with Michael Dettweiler, first of all the completion of the proof of a theorem announced by Fujita 35 years ago. THM 1. If one has a Kaehler family fibred over a curve B, then the direct image V of the relative dualizing sheaf is the direct sum of an ample vector bundle A and of a unitary flat vector bundle W. I shall then describe counterexamples to a question raised by Fujita 32 years ago. Fujita's Question: Is V semi ample? In view of the previous theorem, the question is equivalent to the finiteness of the monodromy representation of the flat bundle W. The examples are based on hypergeometric integrals, and lead to surfaces S which are abelian coverings with group (\ZZ/n)^2 of the Del Pezzo surface Z of degree 5, branched over a union of lines forming a bianticanonical divisor. The Albanese map of S is a fibration onto a curve B of genus b, with fibres of genus g = (n-1)/2, and with g=2b. The fibration is semistable, has only 3 singular fibres, the union of two smooth curves of genus b. The simplest such examples are for n=5, where S is a ball quotient. These coverings, for n=5, were a subset of those considered in my work with I. Bauer, to disprove a conjecture of Enriques on canonical surfaces. Time permitting I shall show how some of these fibrations yield the Shimura curves in M_g classified by Moonen.

Herb Clemens (Ohio State)

A global model for heterotic-F-theory duality with Wilson line symmetry breaking

String theory unifies the the four fundamental forces, gravity, electro-magnitism, strong nuclear and weak nuclear, at very high energy levels. The symmetry of the unified theory is 'broken' as energy levels decrease, into that of the physicists' standard model for the space-time universe we experience. Heterotic F-theory duality models this symmetry breaking in the form of SU(5) symmetry being 'broken down' to SU(3) x SU(2) x U(1) symmetry. This transition is modeled geometrically as the deformation of A4-rational double point surface singularities into A2 and A1 singularities. These are packaged into 'F-theory,' a semi-stable degeneration of an elliptically fibered Calabi-Yau fourfold into the union of two 'DP9'-bundles' meeting transversely along an elliptically fibered Calabi-Yau threefold. By work of Bob Friedman and co-authors John Morgan and Ed Witten, each DP9-bundle corresponds to a flat E8-bundle on the CY-threefold, the makings of a 'heterotic theory.' In joint work with Pantev, Raby and Sheshmani, we produce what is to our knowledge the first global example of this heterotic F-theory duality with symmetry breaking that yields the numerical invariants required by a theory consistent with the physicists' standard model of the observable world.

Ron Donagi (University of Pennsylvania)

Super Riemann surfaces and their moduli

Super Riemann surfaces exhibit many of the familiar features of ordinary curves, and some novelties. They have moduli spaces and Deligne--Mumford compactifications. One can add punctures - but these come in two varieties. (Physicists name these Ramond and Neveu-Schwarz.) I will survey some of the expected and unexpected features, emphasizing a recent proof that for genus g at least 5, the moduli space of super Riemann surfaces is not projected (and in particular is not split): it cannot be holomorphically projected to its underlying reduced manifold. Physically, this means that certain approaches to superstring perturbation theory that are very powerful in low orders have no close analog in higher orders. Mathematically, it means that the moduli space of super Riemann surfaces cannot be constructed in an elementary way starting with the moduli space of ordinary Riemann surfaces. It has a life of its own.

Simon Donaldson (Imperial College)

Adiabiatic limits and co-associative fibrations

A fibration by co-associative submanifolds (with some singular fibres) is an intriguing structure on a 7-manifold with holonomy G2. Important examples have been found by Kovalev, in which the singularities are modelled on Lefschetz pencils. In this lecture we study the geometry of these fibrations, building on earlier work of Gukov, Yau, Zaslow and Barraglia. We consider an "adiabatic limit " which produces simpler equations and set up some of the resulting theory.

Philip Engel (Columbia)

Looijenga's cusp conjecture and mirror symmetry

A cusp singularity is a surface singularity whose minimal resolution is a reduced cycle of smooth rational curves meeting transversely. Cusp singularities come in naturally dual pairs. In 1981, Looijenga proved that whenever a cusp singularity is smoothable, the minimal resolution of the dual cusp is an anticanonical divisor of some smooth rational surface. He conjectured the converse. We will outline a proof of Looijenga's conjecture based on a combinatorial criterion for smoothability given by Friedman and Miranda in 1983. The key step is the construction of a certain punctured 2-sphere endowed with transition functions in the integral-affine transformation group. Such a sphere can be constructed via Symington's surgeries as the base of a Lagrangian torus fibration on a rational surface with the appropriate anticanonical divisor. If time allows, we will discuss the relationship to mirror symmetry and Gross--Hacking--Keel's proof of Looijenga's conjecture, whose ``polytope" construction of a smoothing is Legendre dual to our ``fan" construction.

Phillip Griffiths (IAS)

Limiting Hodge structures and boundaries of moduli

The subject of how polarized Hodge structures degenerate is fairly highly developed. For 1-parameter families the story is well known, but for several parameter families it is less familiar, as is also the case for the introduction of reduced limits of period mappings and the relation to boundary components of moduli spaces of polarized Hodge structures. In this talk we will review the above and illustrate by one example how it may be applied to help understand the boundary of the KSBA moduli space of one particular algebraic surface whose period domain is not Hermitian symmetric.

Joe Harris (Harvard)

Interpolation for curves in projective space

This talk will be concerned with the problem of "curve fitting" in algebraic geometry: specifically, when can we find a curve of given degree and genus passing through r general points in projective r-space? This leads us to another, related problem: when do general points of a curve impose independent conditions on sections of a vector bundle? Recently, Atanasov, Larson and Yang have given an answer to the curve-fitting problem for non-special curves; in this talk I'll report on their work.

Daniel Huybrechts (Universitat Bonn)

The K3 category of a cubic fourfold

According to Kuznetsov, the bounded derived category of a smooth cubic fourfold contains a full subcategory that behaves like the derived category of a K3 surface (and sometimes is equivalent to one). Ideally, one would like to have a theory that parallels the well established one for K3 surfaces and I will explain what is known in this direction.

Radu Laza (Stony Brook)

Birational geometry of the moduli space of hyperelliptic quartic K3s

The study of compactions of the moduli space of K3 surfaces is a problem of great interest. For low degree cases, E. Looijenga has constructed a framework that provides a comparison between the two naturally available compactifications in this case: GIT and Baily--Borel. In this talk, I will discuss an enrichment of this picture, essentially a continuous interpolation between the GIT and BB models. While the discussion will be mostly concerned with the case of hyperelliptic quartic K3 surfaces, we expect such an interpolation to hold quite generally. This is inspired and quite analogous to the Hassett--Keel program that studies the birational geometry of the moduli space of stable curves. This is a report on joint work with K. O'Grady.

Eduard Looijenga (Universiteit Utrecht and Tsinghua University)

The rational homotopy type of compactifications of Baily--Borel and toric type and their MHS

We describe such a homotopy type as the classifying space of a category. For the classical Satake compactification of the moduli space of principally polarized abelian varieties this recovers work of Charney and Lee. In that case we also identify the stable cohomology in geometric terms and determine its mixed Hodge numbers. This is in part joint work with my student Jiaming Chen.

Rick Miranda (Colorado State)

Linear systems on edge-weighted graphs

If G is a connected graph with positive real weights attached to each edge, one can define divisors, linear equivalence, linear systems, etc., following work over the past years begun by Baker and Norine. In joint work with Rodney James, we prove a version of Riemann--Roch in this setting, which seems distinct from the tropical setting. We explore alternative versions of theorems inspired by the theory of algebraic curves. The work shows that a Riemann--Roch theorem is true even for structures on finite sets weaker than edge-weighted graphs.

John Morgan (Simons Center for Geometry and Physics)

Bordism and surgery theory

AbIn this talk, inspired by intersection cohomology, we construct a bordism theory Ω^{PD}_*(X) whose closed objects are stratified spaces with complexes of sheaves satisfying Poincaŕe duality over the integers and whose bordisms are compact stratified spaces with boundary with complexes satisfying Lefschetz duality over the integers. The bordism groups of a point are four-periodic groups: Z in degrees 4k given by the signature and Z/2Z in degrees 4k + 1 given by the deRham invariant. This bordism theory consists of the correct 'probes' for surgery theory in the following sense. A high dimensional surgery problem, that is to say a normal map f : M → N between closed manifolds, is completely determine by the signature and Kervaire invariant obstructions of the induced 'surgery problems' along the bordism elements Ω^{PD}_*(N). These obstructions given multiplicative homomorphisms on the X bordism theory which are equivalent to the homotopy class of the normal map.

David Morrison (UC Santa Barbara)

Birational geometry of Calabi--Yau varieties

I will survey work of Clemens, Friedman and Reid from the late 80's and early 90's on the birational geometry of Calabi--Yau threefolds, discuss interesting connections to decompositions of the derived category of coherent sheaves found by Bondal, Orlov, and Kawamata, and explore prospects for extending our understanding to Calabi--Yau varieties of higher dimension.

Nicholas Shepherd-Barron (King's College London)

Exceptional groups and simultaneous log resolutions of elliptic singularities

For a long time it has been known that deformations of elliptic singularities possess simultaneous log resolutions, in which del Pezzo surfaces appear. In this talk I will explain how this construction also appears naturally in the context of principal bundles over elliptic curves and the reductions of their structure groups. This is a development of previous work by Friedman and Morgan, and by Helmke and Slodowy, and has been done jointly with Ian Grojnowski.

Edward Witten (IAS)

The super period matrix with Ramond punctures

I will describe how the period matrix of a super Riemann surface generalizes some of the classical theory. Then I will explain a generalization of this to include Ramond punctures, and sketch why this is useful for explaining some by now classical results in superstring perturbation theory.