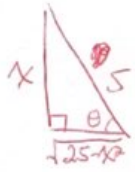
Math 2300-013: Quiz 13

Name: _____ Score: _____

This quiz has TWO questions: One on each side of this paper. Hand this in at the start of class Monday.

1. (8 points) Evaluate the following integrals:

- (3 pts) $\int \sqrt{25-x^2}\,dx$



$5\cos\theta = \sqrt{25-x^2}$

$5\sin\theta = x$

$5\cos\theta\, d\theta = dx$

$= \int 5\cos\theta \cdot 5\cos\theta\, d\theta$

$= 25\int \cos^2\theta\, d\theta$

$= 25\cdot\frac{1}{2}\int (1+\cos 2\theta)\, d\theta$

$= \frac{25}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$

$= \frac{25}{2}(\theta + \sin\theta\cos\theta) + C$

$= \boxed{\frac{25}{2}\left(\arcsin\left(\frac{x}{5}\right) + \frac{x\sqrt{25-x^2}}{25}\right) + C}$

- (3 pts) $\int x^2 \sin(2x)\,dx$ By parts

$u = x^2 \quad du = 2x\,dx$

$dv = \sin(2x)\,dx \quad v = -\frac{1}{2}\cos(2x)$

$= -\frac{x^2}{2}\cos(2x) + \frac{1}{2}\cdot 2\int x\cos(2x)\,dx$

→ By parts, again:

$u = x \quad du = dx$

$dv = \cos(2x)\,dx \quad v = \frac{1}{2}\sin(2x)$

$\int x\cos(2x)\,dx = \frac{x}{2}\sin(2x) - \frac{1}{2}\int \sin(2x)\,dx$

$= \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C$

$= \boxed{-\frac{x^2}{2}\cos(2x) + \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C}$

- (2 pts) $\int_1^2 \frac{x^2}{1+x^3}\,dx$ u-sub:

$u = 1+x^3 \quad du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

If $x=1$, $u=2$

$x=2$, $u=9$

$\rightarrow \frac{1}{3}\int_2^9 u^{-1}\,du = \frac{1}{3}\ln|u|\Big|_2^9 = \frac{1}{3}(\ln(9)-\ln(2)) = \boxed{\ln\left(\sqrt[3]{\frac{9}{2}}\right)}$

2. (2 points) Eliminate the parameter t in the following set of parametric equations to obtain a Cartesian equation relating x and y:

$$x(t) = \sqrt{t+1}$$

$$y(t) = \frac{1}{t}.$$

$t = \frac{1}{y} \rightarrow \boxed{x = \sqrt{\frac{1}{y} + 1}}$

$x^2 - 1 = \frac{1}{y}$

$y = \frac{1}{x^2 - 1}$