Math 2300-013: Quiz 12

Name: ____________________________ Score: __________

This quiz has TWO questions: One one each side of this paper.

1. (5 points) Find a power series representation for this function, centered about $a = 0$.

$$\arctan(x)$$

Solution:

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

2. (5 points) Solve (make sure to write your final answer in the form $y = f(x)$ where f is a function of x):

$$y \cdot y' = x(1 + y^2)$$
$$y(1) = 2$$

Solution:

$$y\frac{dy}{dx} = x(1+y^2)$$

$$\frac{y}{1+y^1}dy = xdx$$

$$\int \frac{y}{1+y^2}dy = \int xdx$$

Use u-sub: $u = 1+y^2$, $\frac{1}{2}du = ydy$

$$\frac{1}{2}\int u^{-1}du = \frac{x^2}{2} + C$$

$$\frac{1}{2}\ln|1+y^2| = \frac{x^2}{2} + C$$

Since $1+y^2$is always positive, drop the abs. value bars:

$$\frac{1}{2}\ln(1+y^2) = \frac{x^2}{2} + C$$

Solve for C:

$$\frac{1}{2}\ln(1+2^2) = \frac{1^2}{2} + C$$

$$\frac{1}{2}\ln(5) - \frac{1}{2} = C$$

Solve for y:

$$\frac{1}{2}\ln(1+y^2) = \frac{x^2}{2} + \frac{1}{2}\ln(5) - \frac{1}{2}$$

$$\ln(1+y^2) = x^2 + \ln(5) - 1$$

Take both sides as powers of e:

$$1+y^2 = e^{x^2+\ln(5)-1}$$

$$y^2 = e^{x^2+\ln(5)-1} - 1$$

$$y = \pm\sqrt{e^{x^2+\ln(5)-1} - 1}$$

Use IC to choose $\pm$:

$$y = \sqrt{e^{x^2+\ln(5)-1} - 1}$$