Math 2300-013: Quiz 10 SOLUTIONS

Name: ____________________ Score: ________

Due Monday 11/4 in class. There is a question on the back - make sure you don't miss it.

1. (a) Find $T_5(x)$, the fifth degree Taylor Polynomial for $f(x) = \ln(x+1)$ at $a = 0$.
Solution: Start with a table to organize the coefficients:

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$\ln(1+x)$	0
1	$\frac{1}{1+x} = (1+x)^{-1}$	1
2	$-(1+x)^{-2}$	-1
3	$2(1+x)^{-3}$	2
4	$-6(1+x)^{-4}$	-6
5	$24(1+x)^{-5}$	24

$$\begin{aligned}
T_5(x) &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(5)}(a)}{5!}(x-a)^5 \\
&= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5 \\
&= 0 + x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5
\end{aligned}$$

So: $T_5(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5$.

(b) Use the polynomial you found in part (a) to estimate the value of $\ln(1.5)$.
Solution:
Note $\ln(1.5) = f(.5)$, so plug $x = .5$ into $T_5(x)$:

$$T_x(.5) = .5 - \frac{1}{2}(.5)^2 + \frac{1}{3}(.5)^3 - \frac{1}{4}(.5)^4 + \frac{1}{5}(.5)^5 \approx 0.407292$$

2. Find the power series representation for $f(x) = \frac{2x}{1+x^2}$. Write you answer in $\sum$-notation.
Solution:
Note the similarities between this and $\frac{a}{1-r}$: Here, the numerator $2x$ is our 'a', and the denominator has $1 - -x^2$, making $-x^2$ our 'r'. Using the geometric series formula:

$$\begin{aligned} \frac{2x}{1+x^2} &= \sum_{n=0}^{\infty}(2x)(-x^2)^n \\ &= 2\sum_{n=0}^{\infty}(-1)^n x^{2n+1} \end{aligned}$$