

# Math 2300: Quiz 6, 10/4/2019

Name: \_\_\_\_\_

Score: \_\_\_\_\_

Please show your work on all questions.

1. (5 points) Suppose you have a point mass of 20 at  $(-1,1)$ , a point mass of 30 at  $(2,2)$  and a point mass of 10 at  $(0,-8)$ . What is the center of mass of this system?

**Solution:**

$$\bar{x} = \frac{20(-1) + 30(2) + 10(0)}{20 + 30 + 10} = \frac{40}{60} = \frac{2}{3}$$

$$\bar{y} = \frac{20(1) + 30(2) + 10(-8)}{20 + 30 + 10} = \frac{0}{60} = 0$$

So the center of mass is at  $(2/3, 0)$ .

2. (5 points) Determine if the following series converges or diverges. Be sure to show your work: State which test you use, and show that the hypotheses of that test are satisfied.

$$\sum_{n=1}^{\infty} \frac{9^n}{n^2}$$

**Solution:**

Consider the divergence test. Look at the limit of the sequence that defines the terms of this series:

$$\lim_{n \rightarrow \infty} \frac{9^n}{n^2}$$

If we try to compute this limit directly, we get an indeterminate form  $\frac{\infty}{\infty}$ . Applying l'Hopital's rule:

$$\lim_{n \rightarrow \infty} \frac{9^n}{n^2} = \lim_{n \rightarrow \infty} \frac{\ln(9)9^n}{2n}$$

We still get  $\frac{\infty}{\infty}$ , the same indeterminate form, so l'Hopital's rule applies again:

$$\lim_{n \rightarrow \infty} \frac{9^n}{n^2} = \lim_{n \rightarrow \infty} \frac{\ln(9)9^n}{2n} = \lim_{n \rightarrow \infty} \frac{\ln(9)^2 9^n}{2} = \infty \neq 0$$

Since  $\lim_{n \rightarrow \infty} \frac{9^n}{n^2} \neq 0$ , by the divergence test the series diverges.