

# Math 2300: Quiz 5, 9/27/2019

Name: \_\_\_\_\_

Score: \_\_\_\_\_

Please show your work on all questions. Note that there is a question on the back!  
Due at the beginning of class Monday 9/30.

1. (4 points) A force of  $F(x) = x^2 - e^{3x}$ , with  $x$  in meters, acts on an object. What is the work required to move the object from  $x = 3$  to  $x = 7$ ?

**Solution:**

$$\begin{aligned} W &= \int_3^7 F(x) dx \\ &= \int_3^7 (x^2 - e^{3x}) dx \\ &= \left( \frac{x^3}{3} - \frac{e^{3x}}{3} \right) \Big|_3^7 \\ &= \left( \frac{7^3}{3} - \frac{e^{3 \cdot 7}}{3} \right) - \left( \frac{3^3}{3} - \frac{e^{3 \cdot 3}}{3} \right) \\ &= \frac{343 - 27 - e^{21} + e^9}{3} \\ &= \frac{316 - e^{21} + e^9}{3} \end{aligned}$$

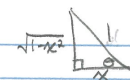
No units on this one, because I forgot to give you units for force ( $F(x)$ ).

2. (2 points) Find the average value of the function  $f(x) = x^4\sqrt{1-x^2}$  over the interval from  $x = 0$  to  $x = 1$ .

**Solution:**

#2

$$\int_{x=0}^{x=1} x^4 \sqrt{1-x^2} dx$$



$$\sin \theta = \sqrt{1-x^2}$$

$$\cos \theta = x$$

$$\cos^4 \theta = x^4$$

$$dx = -\sin \theta d\theta$$

$$\int_{x=0}^{x=1} \cos^4(\theta) \sin \theta (-\sin \theta d\theta)$$

$$= - \int_{x=0}^{x=1} \cos^4(\theta) \sin^2 \theta d\theta$$

$$\begin{aligned} & \text{power reduce} \quad \cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta)) \\ & \sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta)) \end{aligned}$$

$$= - \int_{x=0}^{x=1} \left[ \frac{1}{2}(1 + \cos(2\theta)) \right]^2 \cdot \frac{1}{2} [1 - \cos(2\theta)] d\theta$$

$$= - \frac{1}{8} \int_{x=0}^{x=1} (1 + 2\cos(2\theta) + \cos^2(2\theta))(1 - \cos(2\theta)) d\theta$$

$$= - \frac{1}{8} \int_{x=0}^{x=1} (1 + \cos(2\theta) - \cos^2(2\theta) - \cos^3(2\theta)) d\theta$$

$$= - \frac{1}{8} \int_{x=0}^{x=1} \left[ 1 + \cos(2\theta) - \frac{1}{2}(1 + \cos(4\theta)) - (1 - \sin^2(2\theta)) \cos(2\theta) \right] d\theta$$

$$= - \frac{1}{8} \int_{x=0}^{x=1} \left[ \frac{1}{2} + \cos(2\theta) - \frac{1}{2} \cos(4\theta) + \cos(2\theta) + \sin^2(2\theta) \cos(2\theta) \right] d\theta$$

$$= - \frac{1}{8} \left[ \frac{1}{2} \theta - \frac{1}{8} \sin(4\theta) + \frac{1}{3} \sin^3(2\theta) \right] \Big|_{x=0}^{x=1}$$

$$= \left[ -\frac{1}{16} \theta + \frac{1}{24} (2 \sin(2\theta) \cos(2\theta)) - \frac{1}{24} (2 \sin(\theta) \cos(\theta))^3 \right] \Big|_{x=0}^{x=1}$$

$$= \left[ -\frac{1}{16} \cos^{-1}(x) + \frac{1}{24} (2 \sin \theta \cos \theta) (\cos^2 \theta - \sin^2 \theta) - \frac{1}{24} (\sin^3 \theta \cos^3 \theta) \right] \Big|_{x=0}^{x=1}$$

$$= \left[ -\frac{1}{16} \cos^{-1}(x) + \frac{1}{16} (\sqrt{1-x^2})(x)(x^2 - (1-x^2)) - \frac{1}{24} (\sqrt{1-x^2})^3 x^3 \right] \Big|_{x=0}^{x=1}$$

$$= \left[ -\frac{1}{16}(0) + \frac{1}{16}(0) - \frac{1}{24}(0) \right] - \left[ -\frac{1}{16}\left(\frac{\pi}{2}\right) + \frac{1}{16}(0) - \frac{1}{24}(0) \right]$$

$$= \boxed{\frac{\pi}{32}}$$

3. (4 points) Find the work done pumping molasses out of a 10 foot tall cylindrical tank with a radius of 2 feet. The density of molasses is  $100\text{lbs}/(\text{ft})^3$  (note that this includes gravity!), and the tank is completely full to begin with. Draw at least one picture to accompany your work.

**Solution:**

Note that the shape is a cylinder, so the cross sections are all circles of radius  $r = 2$ . Look at one slice first:

$$\begin{aligned}W_{\text{slice}} &= F * d \\&= (\rho * \text{vol}) * d \\&= (100 * \pi r^2 * dy) * (10 - y) \\&= 400\pi(10 - y)dy\end{aligned}$$

Now integrate it:

$$\begin{aligned}W &= \int_0^{10} 400\pi(10 - y)dy \\&= 400\pi\left(10y - \frac{y^2}{2}\right)\Big|_0^{10} \\&= 400\pi(100 - 50) \\&= 20000\pi\end{aligned}$$

So the solution is  $20000\pi \text{ft} \cdot \text{lbs}$ .