Math 2300-013: Quiz 2, 9/6/2019

Name: ________________________________ Score: _________

Please show your work on all questions.

1. (5 points) Evaluate the following integral:

$$\int \sin^2(x)dx$$

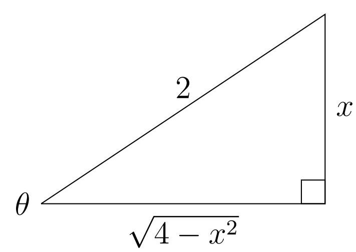
Solution: Use the power reducing formula! $\sin^2(x) = \frac{1-\cos(2x)}{2}$:

$$\begin{aligned}\int \sin^2(x)dx &= \int \frac{1-\cos(2x)}{2}dx \\ &= \frac{1}{2}\int (1-\cos(2x))dx \\ &= \frac{1}{2}(x - \frac{1}{2}\sin(2x)) + C \\ &= \frac{1}{2}x - \frac{1}{4}\sin(2x) + C\end{aligned}$$

2. (5 points) Evaluate the following integral:

$$\int_0^{\pi/4} x^3\sqrt{4-x^2}dx$$

Solution:
Draw a triangle to go with this, and we will use it to go back to x's in the end:



We use $2\cos(\theta) = \sqrt{4-x^2}$. We will also need to use $x = 2\sin(\theta)$.
To get dx in terms of θ, notice $x = 2\sin(\theta)$ gives $dx = 2\cos(\theta)d\theta$.

Substituting in:

$$\int_0^{\pi/4} x^3\sqrt{4-x^2}dx = \int_{x=0}^{x=\pi/4} 2^3\sin^3(\theta)2\cos(\theta)2\cos(\theta)d\theta$$

$$= 2^5\int_{x=0}^{x=\pi/4} \sin^3(\theta)\cos^2(\theta)d\theta$$

Here, we'll do a u-sub with $u = \cos(\theta)$

Using trig identities to get ready:

$$= 2^5\int_{x=0}^{x=\pi/4} \sin^2(\theta)\cos^2(\theta)\sin(\theta)d\theta$$

$$= 2^5\int_{x=0}^{x=\pi/4} (1-\cos^2(\theta))\cos^2(\theta)\sin(\theta)d\theta$$

let $u = \cos(\theta)$, then $du = -\sin(\theta)d\theta$:

$$= -2^5\int_{x=0}^{x=\pi/4} (1-u^2)u^2du$$

$$= -2^5\int_{x=0}^{x=\pi/4} (u^2-u^4)du$$

$$= -2^5\left(\frac{u^3}{3}-\frac{u^5}{5}\right)\Big|_{x=0}^{x=\pi/4}$$

$$= -2^5\left(\frac{(\cos(\theta))^3}{3}-\frac{(\cos(\theta))^5}{5}\right)\Big|_{x=0}^{x=\pi/4}$$

From the triangle:

$$= -2^5\left(\frac{(\frac{\sqrt{4-x^2}}{2})^3}{3}-\frac{(\frac{\sqrt{4-x^2}}{2})^5}{5}\right)\Big|_{x=0}^{x=\pi/4}$$

$$= -2^5\left(\frac{(\sqrt{4-x^2})^3}{3\cdot 2^3}-\frac{(\sqrt{4-x^2})^5}{5\cdot 2^5}\right)\Big|_{x=0}^{x=\pi/4}$$

$$= -2^5\left[\left(\frac{(\sqrt{4-(\pi/2)^2})^3}{3\cdot 2^3}-\frac{(\sqrt{4-(\pi/2)^2})^5}{5\cdot 2^5}\right)-\left(\frac{(\sqrt{4})^3}{3\cdot 2^3}-\frac{(\sqrt{4})^5}{5\cdot 2^5}\right)\right]$$

And we are done!