① $\int e^x \cos(x)\,dx$ → Integration by parts: boomerang

$$\text{let } u = e^x \rightarrow du = e^x dx$$
$$dv = \cos(x)dx \rightarrow v = \sin x$$

$$= e^x \sin x - \int e^x \sin x\,dx \qquad \text{let } u = e^x \rightarrow du = e^x dx$$
$$dv = \sin(x)dx \rightarrow v = -\cos(x)$$

$$\int e^x \cos x\,dx = e^x \sin x - \left(e^x(-\cos x) + \int e^x \cos x\,dx\right)$$
$$\int e^x \cos x\,dx = e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$
$$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x + C$$
$$\int e^x \cos x\,dx = \boxed{\tfrac{1}{2}e^x \sin x + \tfrac{1}{2}e^x \cos(x) + C}$$

② $\int x \ln(x)\,dx$ → Int. by parts

$$\text{let } u = \ln(x) \rightarrow du = \tfrac{1}{x}dx$$
$$dv = x\,dx \rightarrow v = \tfrac{1}{2}x^2$$

$$= \tfrac{1}{2}x^2 \ln(x) - \tfrac{1}{2}\int x^2 \cdot \tfrac{1}{x}dx$$
$$= \tfrac{1}{2}x^2 \ln(x) - \tfrac{1}{2}\int x\,dx$$
$$= \boxed{\tfrac{1}{2}x^2 \ln(x) - \tfrac{1}{4}x^2 + C}$$

③ $\int \cos^3(x)\,dx = \int (1-\sin^2 x)\cos(x)\,dx$ → let $u = \sin(x)$, $du = \cos x\,dx$

$$= \int (1-u^2)du = u - \frac{u^3}{3} + C = \boxed{\sin(x) - \tfrac{1}{3}\sin^3(x) + C}$$

④ $\int \sec^3(x)\,dx$ Int. by parts → $u = \sec x$, $du = \sec x \tan x\,dx$

$$dv = \sec^2 x\,dx, \quad v = \tan(x)$$

$$= \sec x \tan x - \int \sec x \tan^2 x\,dx$$
$$\int \sec^3(x)dx = \sec x \tan x - \int \sec(x)(\sec^2 x - 1)dx$$
$$2\int \sec^3 x\,dx = \sec x \tan x + \int \sec x\,dx$$
$$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan(x)| + C$$
$$\boxed{\int \sec^3 x\,dx = \tfrac{1}{2}\sec x \tan x + \tfrac{1}{2}\ln|\sec x + \tan(x)| + C}$$

⑤ $\int \sin^3 x \cos^2 x\,dx = \int (1-\cos^2 x)\cos^2 x \sin x\,dx$ let $u = \cos x$, $du = -\cos x\,dx$

$$= -\int (1-u^2)u^2 du$$
$$= \int (u^4 - u^2)du = \frac{u^5}{5} - \frac{u^3}{3} + C = \boxed{\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C}$$

⑥ $\int_2^3 \frac{1}{x^2-1}\,dx \rightarrow$ Partial Fractions: $\frac{1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$ clear denominators

$= \int_2^3 \frac{-1/2}{x+1}\,dx + \int_2^3 \frac{1/2}{x-1}\,dx$ $\qquad 1 = A(x-1) + B(x+1)$

$= -\frac{1}{2}\ln(x+1)\Big|_2^3 + \frac{1}{2}\ln(x-1)\Big|_2^3$ $\qquad 1 = x(A+B) + B - A \rightarrow \begin{matrix} A+B=0 \\ A=-B \end{matrix} \quad \begin{matrix} B-A=1 \\ 2B=1 \end{matrix}$

$= -\frac{1}{2}(\ln(4)-\ln(3)) + \frac{1}{2}(\ln(2)-\ln(1))$ $\qquad \rightarrow B = \frac{1}{2},\ A = -1/2$

$= \boxed{-\frac{1}{2}\ln(4) + \frac{1}{2}\ln(3) + \frac{1}{2}\ln(2)}$

⑦ $\int \frac{10}{(x-1)(x^2+9)}\,dx \longrightarrow$ Partial Fractions: $\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$

$\hookrightarrow = \int \frac{1}{x-1}\,dx + \int \frac{-x-1}{x^2+9}\,dx$ $\qquad 10 = A(x^2+9) + (Bx+C)(x-1)$

$= \ln|x-1| + C - \underbrace{\int \frac{x}{x^2+9}\,dx}_{\text{u-sub}} - \underbrace{\int \frac{1}{x^2+9}\,dx}_{\text{arctan integral}}$ $\qquad 10 = Ax^2 + 9A + Bx^2 - Bx + Cx - C$

$10 = x^2(A+B) + x(C-B) + 9A - C$

$A+B=0 \qquad C-B=0 \qquad 9A-C=10$

$A=-B \qquad C=B \qquad A=1$

$A=1, \quad B=-1 \quad C=-1$

$\int \frac{x}{x^2+9}\,dx$ let $u = x^2+9$

$\frac{1}{2}du = x\,dx$

$\downarrow$

$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|x^2+9| + C$

$\int \frac{1}{x^2+9}\,dx = \frac{1}{9}\int \frac{1}{(\frac{x}{3})^2+1}\,dx$

$= \frac{1}{9}\arctan\left(\frac{x}{3}\right)\cdot 3 + C$

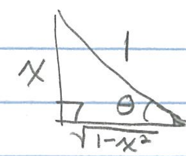
$= \frac{1}{3}\arctan\left(\frac{x}{3}\right)$

$\rightarrow \int \frac{10}{(x-1)(x^2+9)}\,dx = \boxed{\ln|x-1| + \frac{1}{2}\ln|x^2+9| + \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C}$

⑧ $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$ Trig sub. let $x = \sin(\theta)$, $dx = \cos(\theta)\,d\theta$

$= \int \frac{1}{\sin^2(\theta)\sqrt{1-\sin^2\theta}}\cdot\cos(\theta)\,d\theta$

$= \int \frac{1}{\sin^2\theta\sqrt{\cos^2\theta}}\cdot\cos\theta\,d\theta$



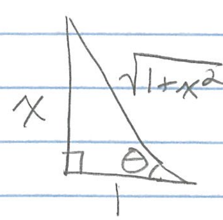
$= \int \frac{1}{\sin^2\theta}\,d\theta = \int \csc^2\theta\,d\theta = -\cot\theta + C$

$= \boxed{\frac{-\sqrt{1-x^2}}{x} + C}$

⑨ $\int \sqrt{1+x^2}\,dx \qquad x = \tan\theta \qquad dx = \sec^2\theta\,d\theta$

$= \int \sqrt{1+\tan^2\theta}\,\sec^2\theta\,d\theta$

$= \int \sqrt{\sec^2\theta}\,\sec^2\theta\,d\theta$



$= \int \sec^3\theta\,d\theta$ We did this one! See #4

$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

$= \boxed{\frac{1}{2}\left(\sqrt{1+x^2}\right)(x) + \frac{1}{2}\ln\left|x + \sqrt{1+x^2}\right| + C}$

⑩ $\int_0^1 \ln(x)\,dx = \lim_{N\to 0}\underbrace{\int_N^1 \ln(x)\,dx}$ → Int. by parts: $u=\ln(x)$ $du=\frac{1}{x}dx$
$dv=dx$ $v=x$

$$=\lim_{N\to 0}\left(x\ln(x)\Big|_N^1-\int_N^1 \tfrac{1}{x}\cdot x\,dx\right)$$

$$=\lim_{N\to 0}\left(-N\ln(N)-\left(x\Big|_N^1\right)\right)$$

$$=\lim_{N\to 0}\left(-N\ln(N)-1+N\right)$$

$$=\lim_{N\to 0}\left(\frac{-\ln(N)}{1/N}\right)-1$$

↖ $\frac{-(-\infty)}{\infty}$ → indeterminate form → l'Hopital's applies

$$=\lim_{N\to 0}\left(\frac{-1/N}{-1/N^2}\right)-1$$

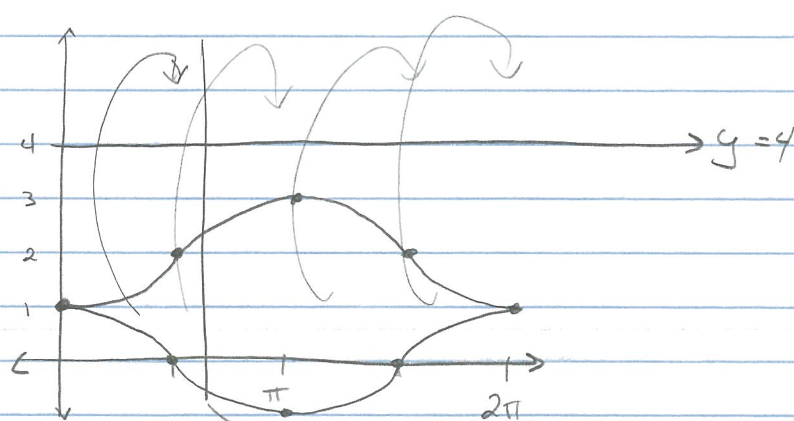
$$=\lim_{N\to 0}(N)-1 = \boxed{-1}$$

⑪ $\int_0^\infty xe^{-x}\,dx$ → integration by parts let $u=x$ $du=1\,dx$
$dv=e^{-x}dx$ $v=-e^{-x}$

↳ $=-xe^{-x}+\int e^{-x}dx$

$=\boxed{-xe^{-x}-e^{-x}+C}$

⑫ $y=\cos(x)$, $y=2-\cos(x)$, $0\le x\le 2\pi$: About $y=4$



Typical cross section:

distance between $y=4$ and $y=2-\cos(x)$
$\Rightarrow r_1=4-(2-\cos(x))$
$=2+\cos(x)$

distance between $y=4$ and $y=\cos(x)$:
$r_2=4-\cos(x)$

$$\boxed{\pi\int_0^{2\pi}\left[\left(4-\cos(x)\right)^2-\left(2+\cos(x)\right)^2\right]dx}$$

(13) Arc length: $\displaystyle\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

$x'(t) = e^t\cos(t) - e^t\sin(t)$

$y'(t) = e^t\sin(t) + e^t\cos(t)$

$$\to \int_0^{\pi}\sqrt{(e^t\cos(t)-e^t\sin(t))^2+(e^t\sin(t)+e^t\cos(t))^2}\,dt$$

$$=\int_0^{\pi}\sqrt{(e^t\cos(t))^2 \cancel{-2e^{2t}\cos(t)\sin(t)} + (e^t\sin(t))^2 + (e^t\sin(t))^2 \cancel{+2e^{2t}\sin(t)\cos(t)} + (e^t\cos(t))^2}\,d$$

$$=\int_0^{\pi}\sqrt{2e^{2t}\cos^2(t)+2e^{2t}\sin^2(t)}\,dt$$

$$=\int_0^{\pi}\sqrt{2e^{2t}(\sin^2(t)+\cos^2(t))}\,dt$$

$$=\sqrt{2}\int_0^{\pi}e^t\,dt = \sqrt{2}\left(e^t\Big|_0^{\pi}\right) = \boxed{\sqrt{2}e^{\pi}-\sqrt{2}}$$

(14) Average Value: $\displaystyle\frac{1}{b-a}\int_a^b f(x)\,dx$

$$\to \frac{1}{\pi-0}\int_0^{\pi}(2\sin x - \sin(2x))\,dx$$

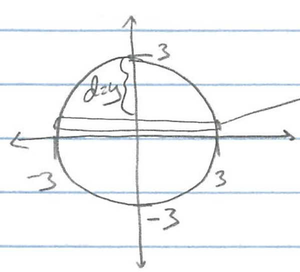
$$=\frac{1}{\pi}\cdot 2\int_0^{\pi}\sin(x)\,dx - \frac{1}{\pi}\int_0^{\pi}\sin(2x)\,dx$$

$$=\frac{2}{\pi}\left(-\cos(x)\Big|_0^{\pi}\right) - \frac{1}{\pi}\left(-\frac{1}{2}\cos(2x)\Big|_0^{\pi}\right)$$

$$=\frac{2}{\pi}(1+1) - \frac{1}{\pi}\left(-\frac{1}{2}\cdot 1+\frac{1}{2}\cdot 1\right)$$

$$=\boxed{4/\pi}$$

(15)



$V_{slice} = \pi r^2\,dy$ radius $= x$ $x^2+y^2=9$

$\to x = \pm\sqrt{9-y^2}$

use positive

$V_{slice} = \pi\left(\sqrt{9-y^2}\right)^2 dy$

$Force_{slice} = \rho\cdot g\cdot$ Volume

$= \pi\rho g\,(9-y^2)\,dy$ $g=9.8$, $\rho=$ density $=1000$

Distance water travels to get out $= 3-y \to Work_{slice} = (3-y)\cdot Force_{slice}$

$$\to W = \int_{-3}^{3}\pi\rho g(9-y^2)(3-y)\,dy$$

$$=\pi\rho g\int_{-3}^{3}(27-9y-3y^2+y^3)\,dy$$

$$=\pi\rho g\left(27y-\frac{9}{2}y^2-y^3+\frac{y^4}{4}\Big|_{-3}^{3}\right.$$

$$=\pi\rho g\left(81-\frac{81}{2}-27+\frac{243}{4}-\left(-81-\frac{81}{2}+27+\frac{243}{4}\right)\right)$$

$$=\pi\rho g\,(162-54) = \pi(1000)(9.8)(108) = \boxed{3{,}325{,}062\text{ J}}$$

(16) Separation of variables: $\frac{dy}{dx} = \frac{xy\sin(x)}{y+1}$

$$\int \frac{(y+1)}{y}\,dy = \int x\sin(x)\,dx$$

$$\int\left(1+\frac{1}{y}\right)dy = \int x\sin x\,dx$$ Integration by parts

$u = x \quad du = dx$

$dv = \sin x\,dx \quad v = -\cos(x)$

$$y + \ln|y| + C = -x\cos(x) + \int \cos(x)\,dx$$

$$y + \ln|y| + C = -x\cos(x) + \sin(x)$$

$y(0) = 1$, so when $x = 0$, $y = 1$:

$$1 + \ln(1) + C = 0 + \sin(0)$$

$$1 + C = 0 \rightarrow C = -1$$

$$\boxed{y + \ln|y| - 1 = -x\cos(x) + \sin(x)}$$ (difficult to solve for y in this case)

(17) $\frac{dy}{dx}\cot^2 x = 1+y \rightarrow \frac{1}{1+y}\,dy = \tan^2 x\,dx$

$$\int \frac{1}{1+y}\,dy = \int \tan^2 x\,dx$$

$$\ln|1+y| + C = \int(\sec^2 x - 1)\,dx$$

$$\ln|1+y| + C = \tan(x) - x \qquad y(\pi/3) = 1 \qquad \tan(\pi/3) = \frac{\sin(\pi/3)}{\cos(\pi/3)} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

$$|1+y| = e^{\tan(x) - x + C}$$

$$|1+y| = C' e^{\tan(x) - x}$$

$$1 + 1 = C' e^{\tan(\pi/3) - \pi/3}$$

$$2 = C' e^{\sqrt{3} - \pi/3}$$

$$2e^{\pi/3 - \sqrt{3}} = C'$$

$$\rightarrow 1+y = 2e^{\pi/3 - \sqrt{3}}\, e^{\tan(x) - x}$$

$$\boxed{y = 2e^{\pi/3 - \sqrt{3}}\, e^{\tan(x) - x} - 1}$$

(19) Newton's Law of Cooling: $\frac{dT}{dt} = k(T - T_s)$

$$\rightarrow \frac{dT}{dt} = k(T - 75)$$

$$\int \frac{1}{T-75}\,dT = \int k\,dt$$

$$\ln|T - 75| = kt + C$$

$$T - 75 = e^{kt + C}$$

positive b/c turkey starts @ 180

$$T = 75 + Ce^{kt}$$

$$T(0) = 180 \rightarrow 75 + C = 180 \rightarrow C = 105$$

$$T(t) = 75 + 105e^{kt}$$

$$T(1/2) = 75 + 105e^{k(\frac{1}{2})} = 150$$

$$75 = 105e^{k(\frac{1}{2})} \rightarrow \ln\left(\frac{75}{105}\right)\cdot 2 = k$$

$$T\left(\frac{3}{4}\right) = 75 + 105e^{(2\ln(75/105))\cdot(3/4)}$$

$$\approx \boxed{138.4°F}$$

20) $P' = .05P - .0005P^2$

$P' = .05P(1 - .01P)$

$P' = .05P\left(1 - \frac{P}{100}\right)$

carrying capacity = $\boxed{100}$

$P'(0) = 0.05\,P(0)\left(1 - \frac{P(0)}{100}\right)$

$= 0.05(10)\left(1 - \frac{10}{100}\right)$

$= .5(.9)$

$= \boxed{.45}$

21) Ratio Test: $\lim_{n\to\infty}\left|\frac{(3x-2)^{n+1}}{(n+1)\cdot 3^{n+1}} \cdot \frac{n\,3^n}{(3x-2)^n}\right| = \lim_{n\to\infty}\left|\frac{(3x-2)n}{(n+1)\cdot 3}\right| = \left|\frac{3x-2}{3}\right|$

Need $\left|\frac{3x-2}{3}\right| < 1 \rightarrow$ $-1 < \frac{3x-2}{3} < 1$

$-3 < 3x-2 < 3$

$-1 < 3x < 5$

$-\frac{1}{3} < x < 5/3$

Check endpoints:

$x = -1/3$: $\sum_{n=1}^{\infty} \frac{(3(-1/3)-2)^n}{n\cdot 3^n} = \sum_{n=1}^{\infty}\frac{(-3)^n}{n(3^n)} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n} \rightarrow$ Converges: alternating harmonic series

$x = 5/3$: $\sum_{n=1}^{\infty}\frac{(3(5/3)-2)^n}{n\cdot 3^n} = \sum_{n=1}^{\infty}\frac{3^n}{n\cdot 3^n} = \sum_{n=1}^{\infty}\frac{1}{n} \rightarrow$ diverges: harmonic series

$\Rightarrow$ Interval of convergence $\boxed{[-1/3, 5/3)}$

22) $f(x) = \frac{x}{1+2x^2} = \frac{x}{1-(-2x^2)} \rightarrow x\sum_{n=0}^{\infty}(-2x^2)^n$

$= x\sum_{n=0}^{\infty}(-1)^n\cdot 2^n\cdot x^{2n} = \boxed{\sum_{n=0}^{\infty}(-1)^n\cdot 2^n\cdot x^{2n+1}}$

23) Converges: geometric w/ $|r| = \frac{1}{\sqrt{2}} < 1$

24) Diverges: divergence test: $\lim_{n\to\infty}\arctan(n) = \pi/2 \neq 0$

25) Diverges: limit comparison test w/ harmonic series

26) Diverges: limit comparison test w/ $\frac{1}{n}$ (notice b/c of the Maclaurin series $\sin\left(\frac{1}{n}\right) = \frac{1}{n} - \frac{1}{6}\cdot\left(\frac{1}{n}\right)^3 + \ldots$

27) Ratio test: $\lim_{n\to\infty}\left|\frac{2^{n+1}(n+1)^3}{(n+1)!}\cdot\frac{n!}{2^n n^3}\right| = \lim_{n\to\infty}\left|\frac{2(n+1)^3}{(n+1)n^3}\right| = \lim_{n\to\infty}\left|\frac{2(n+1)^2}{n^3}\right| = 0$

$\rightarrow$ Converges

28. Converges: limit comparison test w/ $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

29. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{4^n (2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot \frac{\pi^{2n}}{2^{2n}} = \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \left(\frac{\pi}{2}\right)^{2n}}_{\text{recognize as the cosine Maclaurin series}}$

$\hookrightarrow \; = \cos(\pi/2) = 0$

30. $\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{3}{5}\right)^n = \sum_{n=1}^{\infty} (-1)(-1)^{n+1} \cdot \frac{1}{n}\left(\frac{-3}{5}\right)^n = -\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\left(\frac{-3}{5}\right)^n$ ← Recognize as $\ln(1+x)$ Maclaurin serie

$\rightarrow \; = -\ln\left(1 + \left(\frac{-3}{5}\right)\right) = \boxed{-\ln(2/5)}$

31. $\sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n \cdot \frac{1}{n!}$ ← Recognize as Maclaurin Series of e^x

$= \boxed{e^{3/5}}$

32. $\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} \longrightarrow$ $x'(\theta) = -\sin\theta + 2\cos(2\theta)$

$y'(\theta) = \cos(\theta) - 2\sin(2\theta)$

$$\frac{y'(\pi/3)}{x'(\pi/3)} = \frac{\frac{1}{2} - 2\sin(2\pi/3)}{-\sqrt{3}/2 + 2\cos(2\pi/3)} = \frac{-\frac{1}{2} - 2(\sqrt{3}/2)}{-\sqrt{3}/2 - 2(1/2)} \cdot \left(\frac{2}{2}\right)$$

$$= \boxed{\frac{-1 - 2\sqrt{3}}{-\sqrt{3} - 2}}$$

33. $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$ $x'(t) = -2\sin(2t)$

$y'(t) = -\sin(t)$

$$= \frac{\frac{d}{dt}\left(\frac{-\sin(t)}{-2\sin(2t)}\right)}{-2\sin(2t)} = \frac{\frac{d}{dt}\left(\frac{\sin(t)}{2\sin(2t)}\right)}{-2\sin(2t)} = \frac{\frac{(2\sin(2t)\cos(t) - 4\sin(t)\cos(2t))}{(2\sin(2t))^2}}{-2\sin(2t)}$$

$$= \frac{2\sin(2t)\cos(t) - 4\sin(t)\cos(2t)}{-8\sin^3(2t)} = \frac{-\cos(t)}{4\sin^2(2t)} + \frac{\sin(t)\cos(2t)}{2\sin^3(2t)}$$

When is this >0? $\frac{\sin(t)\cos(2t)}{2\sin^3(2t)} > \frac{\cos(t)}{4\sin^2(2t)}$

$$\frac{2\sin(t)\cos(2t)}{\sin(2t)} > \cos(t)$$

$$\frac{2\cancel{\sin(t)}\cos(2t)}{2\cancel{\sin(t)}\cos(t)} > \cos(t)$$

$$\frac{2\cos^2(t) - 1}{\cos(t)} > \cos(t)$$

$(\pi/2, \pi)$

Key: $x = r\cos\theta$ $\quad$ $\tan\theta = \frac{y}{x}$

$y = r\sin\theta$ $\quad$ $x^2+y^2=r^2$

(34) $r = 3\sin\theta$ $\qquad x = r\cos\theta \qquad y = r\sin\theta$

$\frac{r}{3} = \sin\theta \longleftrightarrow \frac{y}{r} = \sin\theta$

$\frac{r}{3} = \frac{y}{r}$

$r^2 = 3y$ $\quad \longrightarrow$ also $x^2+y^2=r^2$, so:

$x^2+y^2 = 3y$

$x^2+y^2-3y+\left(\frac{3}{2}\right)^2 = 0 + \left(\frac{3}{2}\right)^2$

$\boxed{x^2+\left(y-\frac{3}{2}\right)^2 = \frac{9}{4}}$

(35) $r = \tan\theta\sec\theta$

$r = \tan\theta \cdot \frac{1}{\cos\theta}$

$r\cos\theta = \tan\theta$ $\quad \longrightarrow x = r\cos\theta$ and $\tan\theta = \frac{y}{x}$

$x = y/x$

$\boxed{x^2 = y}$

(36) $r = \cos 2\theta \longrightarrow x = r\cos\theta \rightarrow x(\theta) = \cos(2\theta)\cos(\theta)$

$x'(\theta) = -2\cos(\theta)\sin(2\theta) - \cos(2\theta)\sin(\theta)$

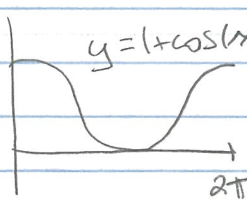
$y = r\sin\theta \rightarrow y(\theta) = \cos(2\theta)\sin(\theta)$

$y'(\theta) = -2\sin(2\theta)\sin\theta + \cos(2\theta)\cos(\theta)$

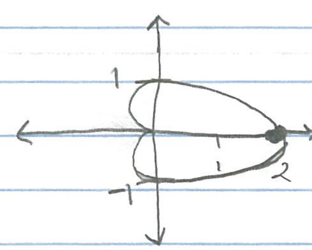
$\frac{dy}{dx} = \frac{y'(\theta)}{x'(\theta)} = \frac{-2\sin(2\theta)\sin\theta + \cos(2\theta)\cos(\theta)}{-2\cos\theta\sin(2\theta) - \cos(2\theta)\sin(\theta)}$ $\quad$ @ $\pi/4$:

$= \frac{-2(1)\left(\frac{\sqrt{2}}{2}\right) + 0\cdot\frac{\sqrt{2}}{2}}{-2\left(\frac{\sqrt{2}}{2}\right)(1) - 0\cdot\frac{\sqrt{2}}{2}} = \boxed{1}$

(37)



$\longrightarrow$



Area $= \frac{1}{2}\int_0^{2\pi}(1+\cos(\theta))^2\,d\theta$

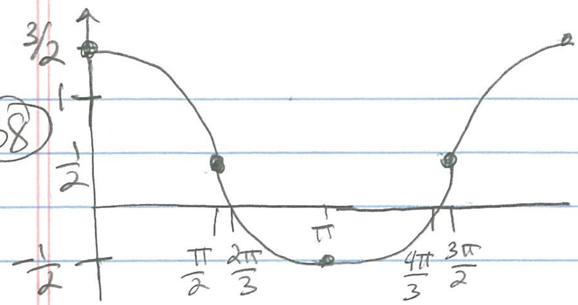
$= \frac{1}{2}\int_0^{2\pi}(1+2\cos\theta+\cos^2\theta)\,d\theta$

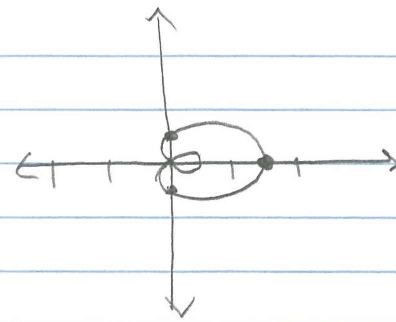
$= \frac{1}{2}\int_0^{2\pi}\left(1+2\cos\theta+\frac{1}{2}(\cos 2\theta+1)\right)d\theta$

$= \frac{1}{2}\left(\theta + 2\sin\theta + \frac{1}{4}\sin(2\theta) + \frac{1}{2}\theta \Big|_0^{2\pi}\right)$

$= \frac{1}{2}(2\pi + 0 + 0 + \pi) = \frac{3\pi}{2}$

(38)





Area = Total area − Inner loop area

$$= \frac{1}{2}\int_0^{2\pi}\left(\frac{1}{2}+\cos\theta\right)^2 d\theta - \frac{1}{2}\int_{2\pi/3}^{4\pi/3}(1/2+\cos\theta)^2 d\theta$$

$$= \frac{1}{2}\int_0^{2\pi}\left(\frac{1}{4}+\cos\theta+\cos^2\theta\right)d\theta - \frac{1}{2}\int_{2\pi/3}^{4\pi/3}\left(\frac{1}{4}+\cos\theta+\cos^2\theta\right)d\theta$$

$$= \frac{1}{2}\int_0^{2\pi}\left(\frac{1}{4}+\cos\theta+\frac{1}{2}\cos(2\theta)+\frac{1}{2}\right)d\theta - \frac{1}{2}\int_{2\pi/3}^{4\pi/3}\left(\frac{3}{4}+\cos\theta+\frac{1}{2}\cos(2\theta)\right)d\theta$$

$$= \frac{1}{2}\left(\frac{3}{4}\theta+\sin\theta+\frac{1}{4}\sin(2\theta)\Big|_0^{2\pi}\right) - \frac{1}{2}\left(\frac{3}{4}\theta+\sin\theta+\frac{1}{4}\sin(2\theta)\Big|_{2\pi/3}^{4\pi/3}\right)$$

$$= \frac{1}{2}\left(\frac{3\pi}{2}\right) - \frac{1}{2}\left(\frac{3}{4}\cdot\frac{4\pi}{3}+\frac{-\sqrt{3}}{2}+\frac{1}{4}\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{3}{4}\cdot\frac{2\pi}{3}+\frac{\sqrt{3}}{2}-\frac{1}{4}\cdot\frac{\sqrt{3}}{2}\right)\right)$$

$$= \frac{3\pi}{4} - \frac{1}{2}\left[\left(\pi-\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{8}\right) - \left(\frac{\pi}{2}+\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{8}\right)\right]$$

$$= \frac{3\pi}{4} - \frac{1}{2}\left[\frac{\pi}{2}-\sqrt{3}+\frac{\sqrt{3}}{4}\right]$$

$$= \frac{\pi}{2}+\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{8} = \boxed{\frac{\pi}{2}+\frac{3\sqrt{3}}{8}}$$

(39) $\int_0^{\pi/3}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$ $\qquad r'(\theta) = 3\cos\theta$

$$= \int_0^{\pi/3}\sqrt{9\sin^2\theta+9\cos^2\theta}\,d\theta = \int_0^{\pi/3}3\,d\theta = 3\left(\frac{\pi}{3}\right) = \pi$$

(40) $r(\theta)=\theta^2 \quad r'(\theta) = 2\theta$

$$\rightarrow \int_0^{2\pi}\sqrt{(\theta^2)^2+(2\theta)^2}\,d\theta = \int_0^{2\pi}\sqrt{\theta^4+4\theta^2}\,d\theta = \int_0^{2\pi}\theta\sqrt{\theta^2+4}\,d\theta$$

let $u=\theta^2+4$, $du = 2\theta d\theta \Rightarrow \frac{1}{2}\int_4^{(2\pi)^2+4}u^{1/2}du = \frac{1}{2}\left(\frac{2}{3}u^{3/2}\Big|_4^{(2\pi)^2+4}\right)$

$$\boxed{\frac{1}{3}\left(\left((2\pi)^2+4\right)^{3/2} - 4^{3/2}\right)}$$