Math 2300: Calculus **Spring 2019**

Lecture 56: Tuesday April 9

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ebAssign due tonight.

56.1 Euler's Method Activity Sheet

One example before the worksheet...

56.1.1 Toy Example/Intro Example

Using the following information, try to estimate the value of a solution function y when $x = 1, 2, 3$.

$$\frac{dy}{dx} = 2x - y$$
$$y(0) = -1$$

We can sketch a slope field to get a general idea of what's going on, and use those little ticks to help us find an estimate.

- First, note that we are "starting" at the point (0,-1). We know this point is on the graph of our solution function. We know that the slope of the graph at this point is $2(0) - (-1) = 1$.
- We can draw the tangent line to the graph at this point using this information. The tangent line is:
$$y = m(x - x_1) + y_1$$
$$y = 1(x - 0) + (-1)$$
$$y = x - 1$$
- We can estimate the value of the solution function at $x = 1$ by using the value of this tangent line:
$$y = 1 - 1 = 0$$
So we imagine that when $x = 1$, the actual solution function value should be close to 0.
- That work gives us an approximation for a second point on our solution function: $(1, 0)$.
- Using the differential equation, we can estimate the slope at this approximate point:
$$\frac{dy}{dx} = 2(0) - 1 = -1$$
- ...So we can keep going and find another tangent line to the curve at this point! This one won't be precise, but it certainly won't be a terrible approximation: it's the best we can do.
Point: $(1, 0)$, slope $= -1$.
Tangent line:
$$y = -1(x - 1) + 0$$
$$y = -x + 1$$

- Let's use this tangent line to get an estimate for $x = 2$:

  $$y = -2 + 1 = -1$$

  Our estimate point is $(2, -1)$.

- Let's get another tangent line. The slope at our most recent point is:

  $$\frac{dy}{dx} = 2(2) - (-1) = 5$$

  Tangent line:

  $$y = 5(x - 2) + (-1)$$
  $$y = 5x - 11$$

- Use the tangent line to get an estimate at $x = 3$:

  $$y = 5(3) - 11 = 4$$

  This gives us an approximate point $(3, 4)$.

We did it! That's Euler's method!

56.1.2 Summary of Process

(More details on this will come in the activity packet...)

(1) Start with an initial value: a point we know is on the solution function. Use the differential equation to get a slope at that point, and thus a tangent line equation.

(2) Use that tangent line equation to get another point.

(3) Repeat!