SOLUTIONS

Slope Fields

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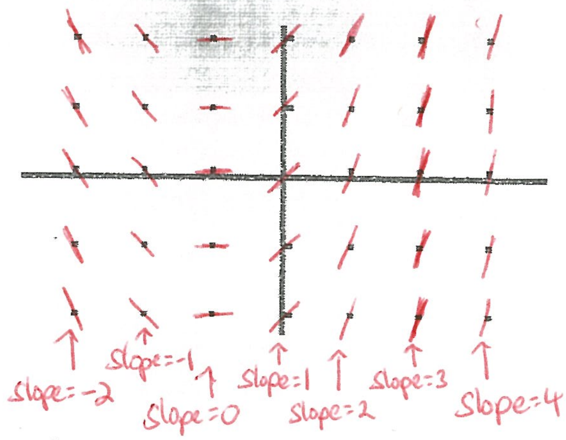
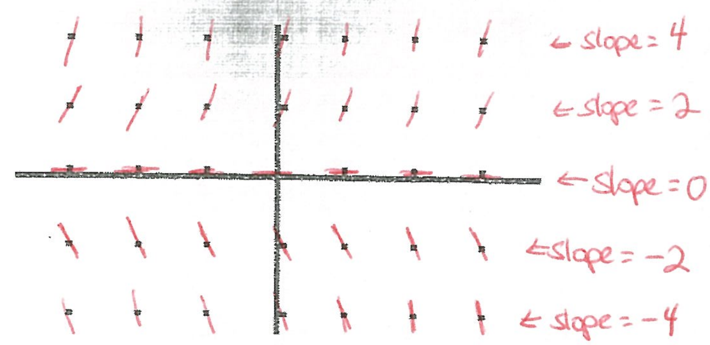
Draw a slope field for each of the following differential equations. Each tick mark is one unit.

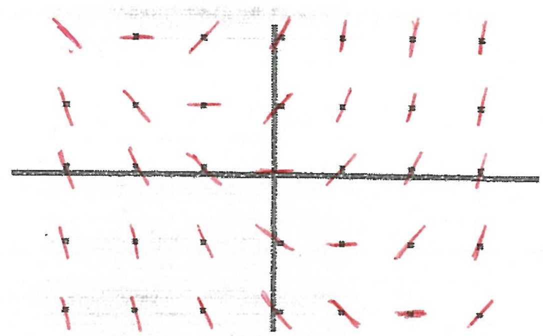
1. $\frac{dy}{dx} = x + 1$ ← only depends on x



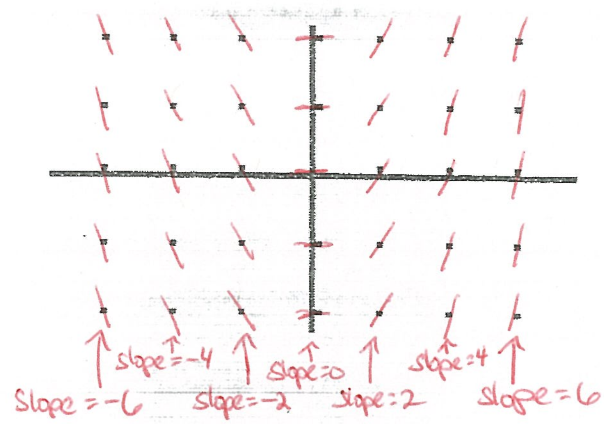
2. $\frac{dy}{dx} = 2y$ ← only depends on y



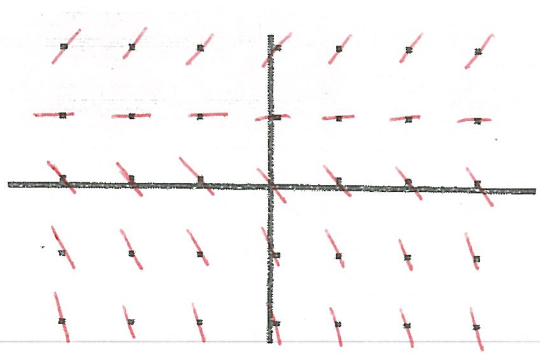
3. $\frac{dy}{dx} = x + y$ ← depends on (x, y) (both!)



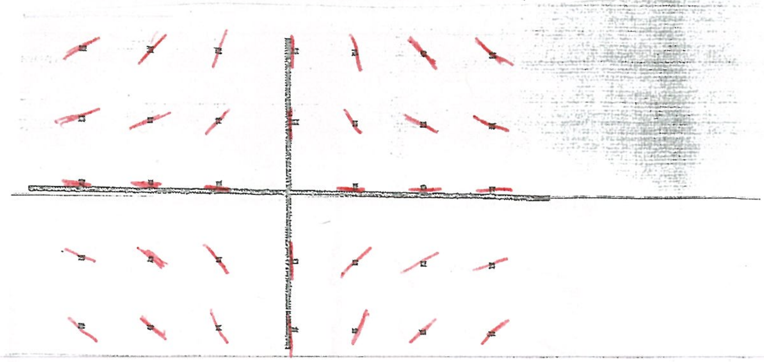
4. $\frac{dy}{dx} = 2x$ ← only depends on x



5. $\frac{dy}{dx} = y - 1$

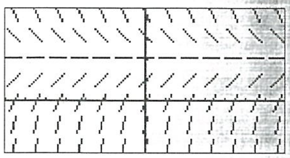


6. $\frac{dy}{dx} = -\frac{y}{x}$ when x = 0 → vertical line



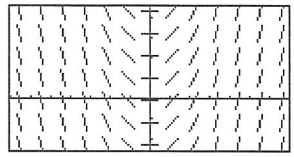
Match the slope fields with their differential equations.

(A)



changes w/ y

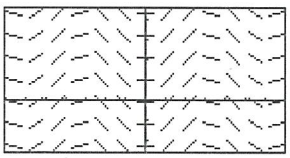
(B)



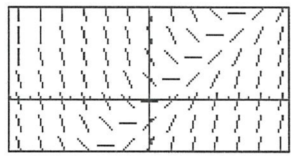
changes w/ x

(C)

looks like $-\cos(x)$



(D)



we know $y = -\cos(x) + C$ is solution

only depends on y

only depends on x

7. $\dfrac{dy}{dx} = \sin x$ C

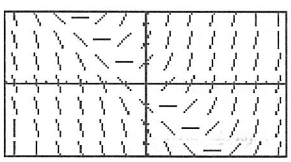
8. $\dfrac{dy}{dx} = x - y$ D

9. $\dfrac{dy}{dx} = 2 - y$ A

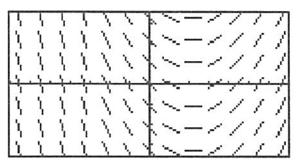
10. $\dfrac{dy}{dx} = x$ B

Match the slope fields with their differential equations.

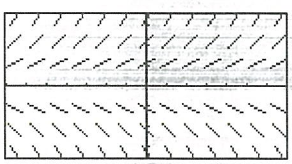
(A)



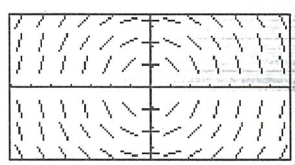
(B)



(C)



(D)



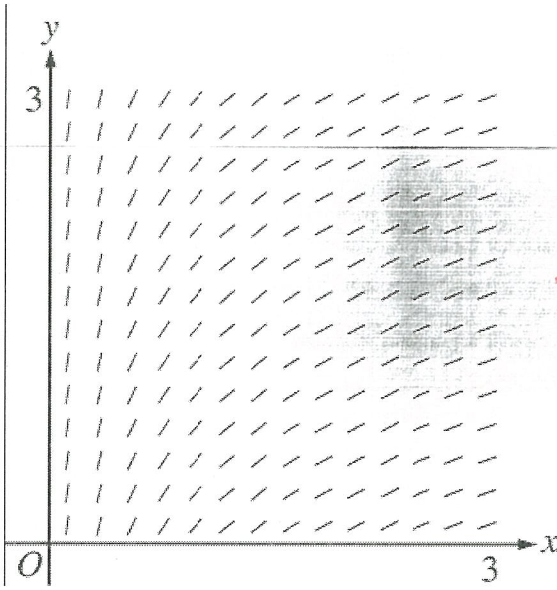
11. $\dfrac{dy}{dx} = 0.5x - 1$ B

12. $\dfrac{dy}{dx} = 0.5y$ C

13. $\dfrac{dy}{dx} = -\dfrac{x}{y}$ D

14. $\dfrac{dy}{dx} = x + y$ A

From the May 2008 *AP Calculus Course Description:*
15.



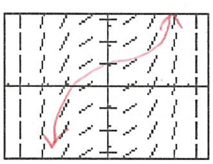
← only depends on x, positive in $x \in [0,3]$

The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$

$\frac{dy}{dx} = 2x$ $\frac{dy}{dx} = e^x = y$ $\frac{dy}{dx} = -e^{-x} = -y$ $\frac{dy}{dx} = -\sin(x)$ $\frac{dy}{dx} = \frac{1}{x}$

16.



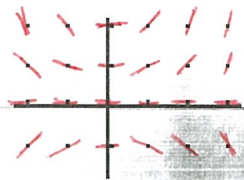
"Connect the lines"

The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = \sin x$ (B) $y = \cos x$ (C) $y = x^2$ (D) $y = \frac{1}{6}x^3$ (E) $y = \ln x$

17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(A) On the axes provided, sketch a slope field for the given differential equation.



(B) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, 1). Then use your tangent line equation to estimate the value of $f(1.2)$. @ (1,1), slope $= \frac{1}{2}$ → Tangent line: $\boxed{y = \frac{1}{2}(x-1)+1}$

Plug in $x = 1.2$: $y = \frac{1}{2}(.2)+1 = \boxed{1.1}$

DO wed.

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(D) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part

(E) Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.

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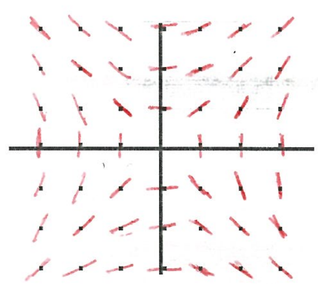
18. Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

(A) On the axes provided, sketch a slope field for the given differential equation.



(B) Sketch a solution curve that passes through the point (0, 1) on your slope field.

do wed.

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 1$.

(D) Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

(E) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.