Math 2300: Calculus **Spring 2019**

Lecture 54: Friday April 5

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54.1 Warm Up

54.2 Introduction to Differential Equations

A **Differential Equation** is any equation that has a derivative in it. For example:

$$\frac{dy}{dx} = x^2 + 1$$

$$y'' + y' = 2y + x$$

When we talk about 'solving' a differential equation, we mean finding an equation y that satisfies the differential equation. For example, the very familiar differential equation:

$$\frac{dy}{dx} = 2x$$

has a family of solutions: $y = x^2 + C$. This one is pretty straight forward. But think about how it can get more complicated:

$$\frac{dy}{dx} = y^2 - y.$$

It's much harder to find a y that satisfies this complicated relationship with its derivative.
In general, some differential equations are hard to solve and some are easy. Some classes of differential equations are still being studied today (people get PhDs in differential equations). So the ones we study in this class are going to be carefully chosen.

54.2.1 Methods To Solve Diff EQ's

To emphasize that the "solution" to a differential equation is a function, I will try to write "solution-function" instead of just saying "solution".

54.2.1.1 Analytic Solutions

We get analytic solutions in rare cases like $y' = 2x$: Where we have a clear family of solution-functions using anti-derivatives.
One technique helps make these cases more obvious **- separation of variables**. We will learn this method in more depth.

54.2.1.2 Graphical Solutions

You can sometimes draw an approximate solution function, even if you don't know the exact function you're drawing. This is at least useful in getting an intuition for what a solution should look like. The technique that we will learn here is **slope fields**. More on this later.

54.2.1.3 Numerical Solutions

You can get values of an approximate solution function, even if you can't get the actual function (the analytic solution). The technique that we will learn here is **Euler's method**.

54.2.2 Slope Fields

Since pictures are nice, let's start with slope fields. We can start with a differential equation that we don't know the answer to, but that might not be too tricky. To define a slope field, let's take an example:

54.2.2.1 Example

Consider the differential equation:

$$y' = y - x.$$

The 'solution' is a function $y(x) = \dots$, but it's not clear what this function y should be. We don't know what the domain or range will be...but we do know that if (x, y) is a point on this function's graph, then the slope at that point is $y - x$. What we can do is take a sample of the $x - y$ plane, and sketch in what the slopes would be at those points.

Draw a blank grid, say from $x = -3$ to $x = 3$, and from $y = -3$ to $y = 3$. This is a lot of points! We can go to each point and draw a tiny line segment (try to just stay on that point) with slope $y - x$. For example, if we go to the point $(2, 3)$, the slope we should draw is $3 - 2 = 1$.

Let's take some time and do this: