Math 2300: Calculus **Spring 2019**

Lecture 32: Wednesday February 27

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32.1 8.3: The Comparison Test

Some series we can immediately recognize as converging or diverging by using one of our familiar tests. For example:

$$\sum_{i=1}^{\infty} \frac{1}{3^{i-1}}$$

converges since it is a geometric series with $r = \frac{1}{3} < 1$. But what about:

$$\sum_{i=1}^{\infty} \frac{1}{3^{i-1} + i}$$

It's not a geometric series, but it's related to it. Keeping in mind that the terms of that geometric series look quite like the terms of this series, we might be able to conclude convergence or divergence. For an individual i:

$$i = 1\text{: } \frac{1}{1} > \frac{1}{1+1}$$

$$i = 2\text{: } \frac{1}{3} > \frac{1}{3+1}$$

$$i = 3\text{: } \frac{1}{9} > \frac{1}{9+1}$$

We see that the terms of our new series are smaller than the terms of the geometric series that we know converges. So our series **should** converge. Let's learn how to rigorously prove this. (Handout project. Make sure everyone fills out the Comparison Test box on the first page correctly. If they get to limit comparison test, mention the notes below. If not, do this at the beginning of next class.)

32.1.1 Limit Comparison Test

**Note: There are actually two valid phrasings.
In both of these phrasings, we require $a_n, b_n \geq 0$!!!

1. Phrasing 1: Suppose a_n and b_n are non-negative sequences:
   - If $\sum b_n$ converges and $\lim\limits_{n\to\infty} \frac{a_n}{b_n} < \infty$, then $\sum a_n$ converges.
   - If $\sum b_n$ diverges and $\lim\limits_{n\to\infty} \frac{a_n}{b_n} \neq 0$, then $\sum a_n$ diverges.
2. Suppose a_n and b_n are non-negative sequences: If $0 < \lim\limits_{n} \to \infty \frac{a_n}{b_n} < \infty$, then $\sum a_n$ converges if and only if $\sum b_n$ converges.