Math 2300: Calculus **Spring 2019**

Lecture 27: Wednesday February 20

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27.1 Series

27.1.1 Recall Notation

If you want to write out a sum:

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

you may want to save space. To do this, you can use $\sum$ notation:

$$\sum_{i=1}^{1} 0i = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

What about if the thing you're adding up is more complicated? Just figure out how to use that "counter" i to your advantage:

$$\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} = \sum_{i=2}^{5} \frac{i}{i+1}.$$

We can also use this to add up the terms of a sequence:

$$a_1 + a_2 + \cdots + a_n = \sum_{i=1}^{n}$$

Recall that our sequences can be infinite. How do we add them up in this case?
Take a sequence $a_1, a_2, ...$ Adding up all of the (infinitely many) terms gives us an (infinite) **series**:

$$\sum_{i=1}^{\infty} a_i = a_1 + a_2 + \cdots$$

Notice the difference between a sequence and a series. In a sequence, we just list all the terms. In a series, we add them all up.
How do we do this infinitely many times? **Limits**. The value of an infinite series is **defined** by its **partial sums**:

$$\sum_{i=1}^{\infty} a_i = \lim_{N \to \infty} \sum_{i=1}^{N} a_i$$

- If $\sum_{i=1}^{\infty} a_i$, we say the series **converges**.
- If $\sum_{i=1}^{\infty} a_i$ is infinite or DNE, we say the series **diverges**.
- We can think of this geometrically by imagining an infinite series as the integral of a step function (draw).

See Faan Tone's Notes for continuation.