Math 2300: Calculus **Spring 2019**

Lecture 19: Friday February 8

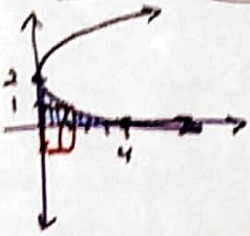
Lecturer: Sarah Arpin

WebAssign due tonight

19.1 Volumes of Solids of Revolution by Cylindrical Shells Part 1 Part II

19.1.1 Example 1

Determine the volume of the solid obtained by rotating the region bounded by $x = (y-2)^2$, the x-axis and the y-axis about the x-axis.



$0 = (y-2)^2 \iff y = 2$

$x = (0-2)^2 \iff x = 4$

Radius of cylinder = height of parabola = y

height of cylinder = x-coord of parabola $= (y-2)^2$

Area of Cylinder = $2\pi r \cdot h$

$\rightarrow V = \int_0^2 2\pi (y)(y-2)^2 = 2\pi \int_0^2 (y^3 - 4y^2 + 4y)dy = 2\pi \left(\frac{y^4}{4} - \frac{4y^3}{3} + 2y^2 \Big|_0^2 \right) = 2\pi \left(4 - \frac{32}{3} + 8\right)$

$= 2\pi \left(\frac{36}{3} - \frac{32}{3}\right) = \boxed{\frac{8\pi}{3}}$

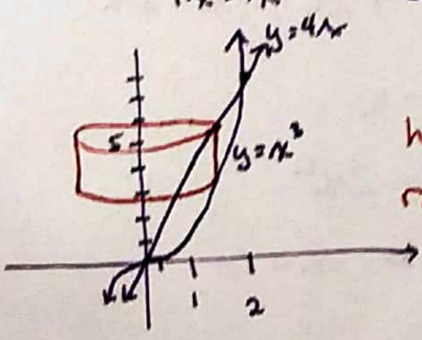
☆ you can check this one w/ washer method: $A = \pi r^2$ and $r = (-\sqrt{x} + 2)$

19.1.2 Example 2

Determine the volume of the solid obtained by rotating the region bounded by $y = 4x$ and $y = x^3$ about the y-axis. For this problem assum $x \geq 0$.

$4x = x^3 \longrightarrow 0 = x(x^2 - 4) = x(x-2)(x+2)$

$x = 0,\ x = 2$



height of cylinder: $4x - x^3$

radius of cylinder: x

Area of Cylinder $= 2\pi r h$

$= 2\pi (4x - x^3)(x)$

$\rightarrow V = 2\pi \int_0^2 (4x^2 - x^4)dx$

$= 2\pi \left(\frac{4x^3}{3} - \frac{x^5}{5} \Big|_0^2 \right)$

$= 2\pi \left(\frac{32}{3} - \frac{32}{5} \right)$

$= 2\pi \left(\frac{160 - 96}{15} \right) = 2\pi \left(\frac{64}{15} \right)$

$= \boxed{\frac{128\pi}{15}}$

19.1.3 Example 3

Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 6x + 9$ and $y = -x^2 + 6x - 1$ about the line $x = 8$.

$(x-3)^2$

Intersect: $x^2 - 6x + 9 = -x^2 + 6x - 1$

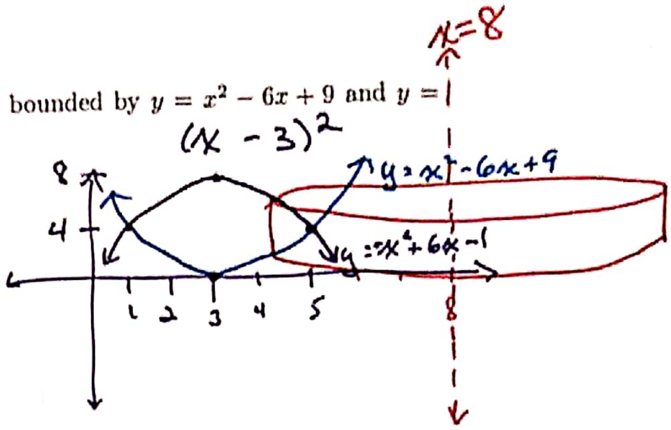
$2x^2 - 12x + 10 = 0$

$x^2 - 6x + 5 = 0$

$(x-1)(x-5) = 0$

$x = 1, \quad x = 5$

$(1,4) \quad (5,4)$



vertex of $y = -x^2 + 6x - 1$

$x = \frac{-b}{2a} = \frac{-6}{-2} = 3$

$(3, 8)$

Radius of cylinder: $8 - x$

Height of cylinder: $(-x^2 + 6x - 1) - (x^2 - 6x + 9)$

$= -2x^2 + 12x - 10$

$V = 2\pi \int_1^5 (8-x)(-2x^2 + 12x - 10)\,dx$

$= 4\pi \int_1^5 (8-x)(-x^2 + 12x - 10)\,dx$

$= 4\pi \int_1^5 (x^3 - 20x^2 + 106x - 80)\,dx$

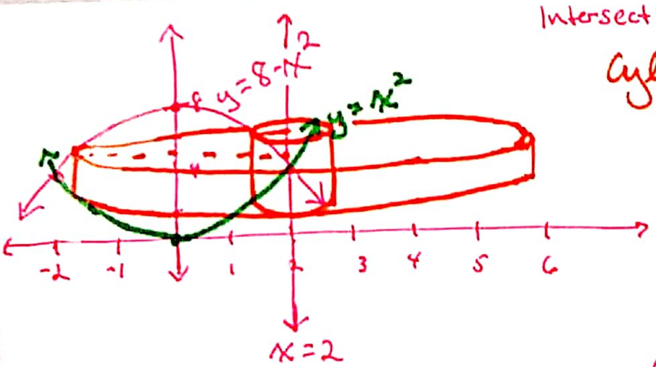
$4\pi \left(\frac{x^4}{4} - \frac{20x^3}{3} + 53x^2 - 80x \Big|_1^5 \right)$

$= 4\pi \left(\frac{5^4}{4} - \frac{4 \cdot 5^4}{3} + 53(5^2) - 80(5) \right) - 4\pi \left(\frac{1}{4} - \frac{20}{3} + 53 - 80 \right)$

$\approx$

19.1.4 Example 4

Determine the volume of the solid obtained by rotating the region bounded by $y = 8 - x^2$, $y = x^2$ about the line $x = 2$.



Intersect: $8 - x^2 = x^2 \rightarrow 4 = x^2 \rightarrow \pm 2 = x,\ y = 4$

Cylinder: Height $= 8 - x^2 - x^2 = 8 - 2x^2$

Radius $= 2 - x$, as x goes from -2 to 2 (see picture – always go from Left to Right)

Area $= 2\pi r h = 2\pi(2-x)(8-2x^2)$

$\rightarrow V = 2\pi \int_{-2}^{2} (2-x)(8-2x^2)\,dx$

$= 2\pi \int_{-2}^{2} (2x^3 - 4x^2 - 8x + 16)\,dx$

$= 2\pi \left(\frac{x^4}{2} - \frac{4x^3}{3} - 4x^2 + 16x \right) \Big|_{-2}^{2}$

$= 2\pi \left(\left(8 - \frac{32}{3} - 16 + 32\right) - \left(8 + \frac{32}{3} - 16 - 32\right) \right)$

$= 2\pi \left(24 - \frac{32}{3} + 40 - \frac{32}{3} \right) = 2\pi \left(64 - \frac{64}{3} \right) = \boxed{\frac{256\pi}{3}}$