Math 2300: Calculus	Spring 2019

Lecture 16: Tuesday February 5

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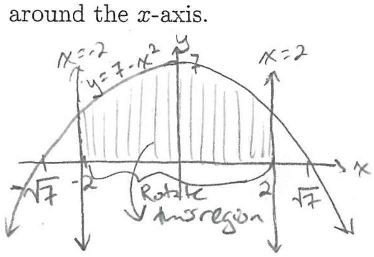
WebAssign due tonight

16.1 Volumes of Solids by Cross Sections

(First, do the worksheet on solids with known cross sections).

16.1.1 Example 1:

Find the volume obtained by rotating the region bounded by $y = 7 - x^2, x = -2, x = 2$, and the x-axis around the x-axis.



The vertical cross-sections of the volume are discs.

Their areas are: $\pi r^2 = \pi(7-x^2)^2$

Integrate this from left to right:

$$\int_{-2}^{2} \pi(7-x^2)^2\,dx = \pi \int_{-2}^{2} (49 - 14x^2 + x^4)\,dx$$

$$= \pi\left(49x - \frac{14x^3}{3} + \frac{x^5}{5}\right)\Big|_{-2}^{2}$$

$$= \pi\left(98 - \frac{14\cdot 8}{3} + \frac{32}{5} - \left(-98 + \frac{14\cdot 8}{3} - \frac{32}{5}\right)\right)$$

$$= \pi\left(196 - \frac{104}{3} + \frac{64}{5}\right) = \boxed{\frac{2612\pi}{15}}$$

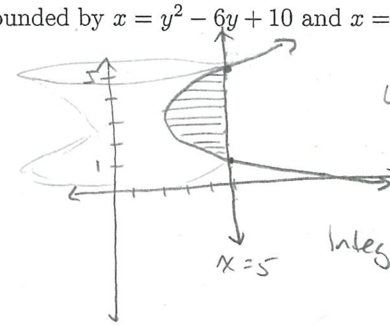
16.1.2 Example 2:

Find the volume obtained by rotating the region bounded by $x = y^2 - 6y + 10$ and $x = 5$ around the y-axis.

Find intersection pts:

$5 = y^2 - 6y + 10$
$0 = y^2 - 6y + 5$
$0 = (y-5)(y-1)$
$y = 5,\ y = 1$
$(5,5),\ (5,1)$

Does parabola cross y-axis?
$0 = y^2 - 6y + 10$
$y = \frac{6 \pm \sqrt{36-40}}{2}$ ← imaginary ⇒ No



Horizontal cross sections will be washers.
Inner radius $= y^2 - 6y + 10$
Outer radius $= 5$
Area $= \pi(5)^2 - \pi(y^2-6y+10)^2$

Integrate from bottom to top:

$$\int_1^5 \pi(25 - (y^2-6y+10)^2)\,dy = \int_1^5 \pi(25 - (y^4 - 12y^3 + 56y^2 - 120y + 100))\,dy = \pi\int_1^5 (-y^4 + 12y^3 - 56y^2 + 120y - 75)\,dy$$

$$= \pi\left(\frac{-y^5}{5} + 3y^4 - \frac{56y^3}{3} + 60y^2 - 75y\Big|_1^5\right) = \boxed{\frac{1088\pi}{15}}$$

$(y^2 - 6y + 10)(y^2 - 6y + 10)$
$y^4 - 12y^3 + 56y^2 - 120y + 100$

16.1.3 Example 3:

Find the volume obtained by rotating the region bounded by $y = 2x^2$ and $y = x^3$ around the x-axis.

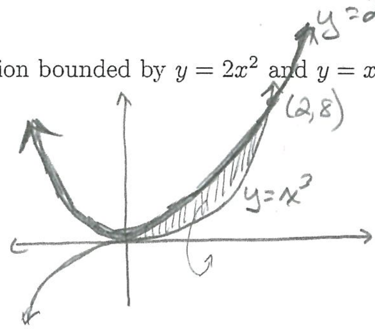
Intersection? $2x^2 = x^3$

$0 = x^3 - 2x^2$

$0 = x^2(x-2)$

$x = 0,\ x = 2$

$(0,0)\quad (2,8)$



Vertical cross sections are washers.

Inner radius: x^3

Outer radius: $2x^2$

Area: $\pi(2x^2)^2 - \pi(x^3)^2$

$$\rightarrow \int_0^2 \pi(4x^4 - x^6)\,dx = \pi\int_0^2 (4x^4 - x^6)\,dx = \pi\left(\frac{4x^5}{5} - \frac{x^7}{7}\Big|_0^2\right)$$

$$= \pi\left(\frac{128}{5} - \frac{128}{7}\right)$$

$$= \pi\left(\frac{7(128) - 5(128)}{35}\right) = \boxed{\frac{256\pi}{35}}$$

16.1.4 Example 4:

Find the volume obtained by rotating the region bounded by $y = 10 - 6x + x^2, y = -10 + 6x - x^2, x = 1$ and $x = 5$ around the line $y = 8$.

$-(10-6x+x^2) = -10+6x-x^2$, so we have a reflection over the x-axis.

Intersection?

$10 - 6x + x^2 = -10 + 6x - x^2$

$2x^2 - 12x + 20 = 0$

$x^2 - 6x + 10 = 0$

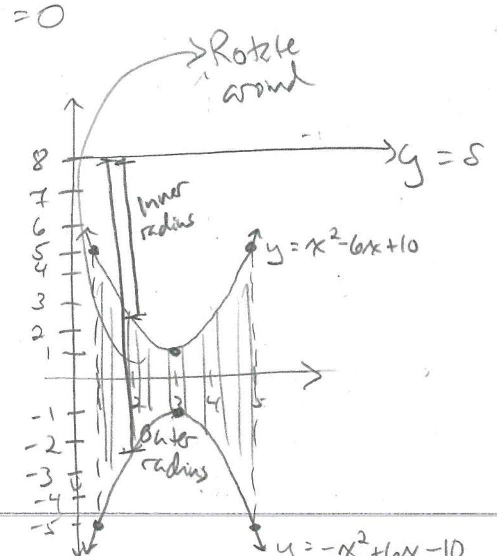
No real solutions

Vertex: $x = \frac{-b}{2a} = \frac{+6}{2} = 3$

$10 - 18 + 9 = 10 - 9 = 1$

$y = 10 - 6 + 1 = 5$

$y = 10 - 30 + 25 = 5$



The vertical cross sections are washers.

Inner radius $= 8 - (x^2 - 6x + 10)$

Outer radius $= 8 + (-x^2 + 6x - 10)$

Area: $\pi(x^2 - 6x + 18)^2 - \pi(-x^2 + 6x - 2)^2$

Integrate:

$$\int_1^5 \pi\left((x^2-6x+18)^2 - (-x^2+6x-2)^2\right)dx$$

$$= \pi\int_1^5 (x^4 - 12x^3 + 72x^2 - 216x + 324) - (x^4 - 12x^3 + 40x^2 - 24x + 4)\,dx$$

$$= \pi\int_1^5 (32x^2 - 192x + 320)\,dx$$

$$= \boxed{\frac{896\pi}{3}}$$