Math 2300: Calculus **Spring 2019**

Lecture 2: Tuesday January 15

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2.1 Day 2: u Substitution Review

2.1.1 Formula

Remember that if you have a composition of functions, you can use u-substitution to make integration easier. In general:

$$\int f(g(x))g'(x)dx$$

can be evaluated by setting $u = g(x)$, which means that $du = g'(x)dx$, and then we can substitute this information in:

$$\int f(g(x))g'(x)dx = \int f(u)du$$

and this latter expression is usually easier to integrate.

2.1.2 To Identify u-substitution

Look for composition of functions! This is the key.

2.1.3 Indefinite Integrals with u-sub

There are a few steps when we have **indefinite integrals**:

1. Can you evaluate the integral? Not all integrals require u-sub. Try to evaluate the integral first!
2. Do you see a composition of functions, where the derivative of the inner function basically appears elsewhere in the integral? For example:

$$\int \sin(x^4)(4x^3)dx$$

   We can clearly see that there is an outer function, the sin function, and an inner function, x^4, and the derivative of the inner function, $4x^3$, appears outside of this composition.
3. Let u represent the inner function.
4. Solve for du by computing $\frac{du}{dx}$ from your let statement with u. This step is crucial - du is not always exactly what you need it to be!
5. Make the necessary adjustments to your du so that you can substitute in the integral and replace dx. We don't have to do this in this example, but we will in the next!

$$\int \sin(x^4)(4x^3)dx$$

$$u = x^4,\, du = 4x^3dx$$
$$\int \sin(x^4)(4x^3)dx = \int \sin(u)du$$

6. Now, integrate!
$$\int \sin(u)du = -\cos(u) + C$$

7. ...and plug back in for u:
$$\cdots = \cos(x^4) + C$$

Now, look at a slightly more complicated example, where we have to make the adjustments to the du, as discussed above:

$$\int \sin(x^4)x^3dx$$
$$u = x^4,\, du = 4x^3dx$$

Oh no! du isn't exactly equal to what we want to replace...that's okay! We can manipulate this expression, as long as we're only moving around constants:

$$du = 4x^3dx \Rightarrow \frac{1}{4}du = x^3dx$$

Now we can substitute in and continue:

$$\begin{aligned}\int \sin(x^4)x^3dx &= \int \sin(u)\frac{1}{4}du \\ &= \frac{1}{4}\int \sin(u)du \\ &= \frac{1}{4}(-\cos(u)) + C \\ &= \frac{-1}{4}\cos(x^4) + C\end{aligned}$$

2.1.4 Definite integrals with u-sub

Similar process! But with one extra step, and you don't have to go back to x's at the end. This is how I like to do it... you may have learned a different way. If you're more comfortable with that way and you're getting right answers, keep doing it your way! If you're more comfortable with the other way, but you're not getting right answers...time to switch!

Here's the process:

1. Can you evaluate the integral? If yes, do so and skip the rest of these steps!
2. Identify a composition of functions, where you see some constant multiple of the derivative of the inner function elsewhere in the integral. Let's continue with our example, but make it a definite integral:
$$\int_{-2}^{1} \sin(x^4)x^3dx$$
3. Let u represent the inner function.
$$u = x^4$$

4. Solve for du by computing $\frac{du}{dx}$ from your let statement with u from the previous step.

$$du = 4x^3 dx$$

5. **Find new bounds of integration:** If $u = g(x)$, then find u when $x =$ (lower bound) to get your new lower bound, and likewise for the upper bound.

$$\text{If } x = -2\text{, then } u = (-2)^4 = 16$$

$$\text{If } x = 1\text{, then } u = (1)^4 = 1$$

6. Making the necessary adjustments to the du expression, we can plug all of this in:

$$\frac{1}{4}du = x^3 dx$$

$$\int_{-2}^{1} \sin(x^4)x^3 dx = \int_{16}^{1} \sin(u)\frac{1}{4}du = \frac{1}{4}\int_{16}^{1} \sin(u)du$$

7. Now integrate! **There's no need to go back to x's, since everything is in terms of u now.**

$$\begin{aligned}\int_{-2}^{1} \sin(x^4)x^3 dx &= \frac{1}{4}\int_{16}^{1} \sin(u)du \\ &= \frac{1}{4}(-\cos(u)|_{16}^{1}) \\ &= \frac{1}{4}(-\cos(1) - -\cos(16)) \\ &\approx -0.37449\end{aligned}$$