

Lecture: Section 6.1: More on area

*Lecturer: Sarah Arpin***Today's Goal:**

Logistics: Our final exam is 7:30am - 10am Wednesday December 9th. Set your phone alarms NOW. Note that this is *AM*.

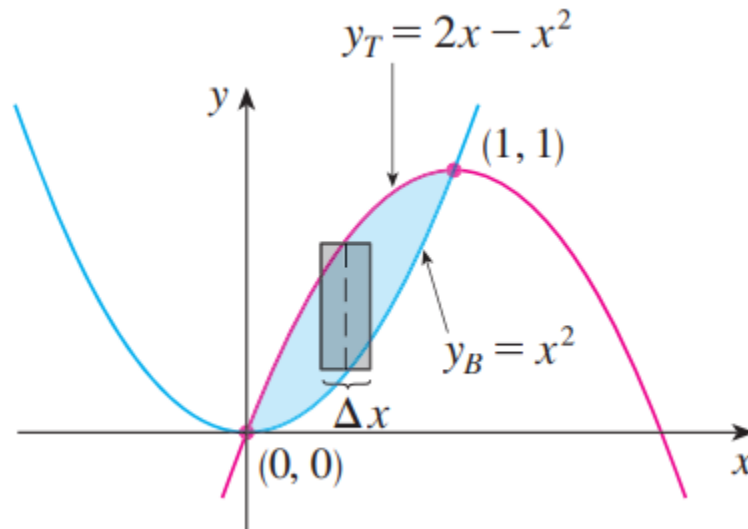
Warm-Up 1.1 Use geometry to find the area enclosed by the curves $y = 2x$, $y = 1$, $x = 1$, and $x = 2$.
Hint: A sketch will help.

- (a) 3
- (b) 2
- (c) 1
- (d) 0
- (e) None of the above.

1.1 Area between two curves

We can use integrals to compute the area between a curve and the x -axis.

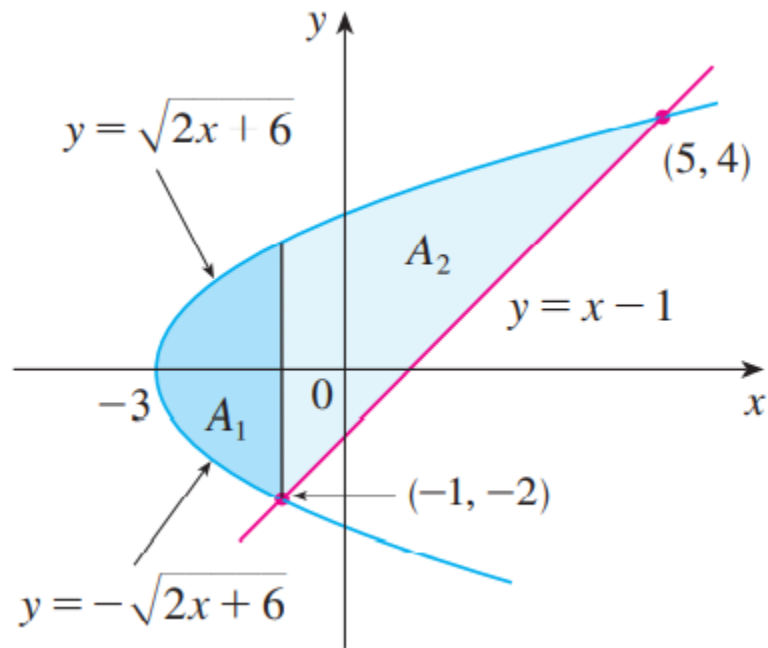
We would like to extend the reach of this method: How can we compute an area that is bounded by curves, not just between a curve and the x -axis?

1.1.1 With respect to x 

Example 1.2 Find the area between the two curves $y = 2x - x^2$ and $y = x^2$ between $x = 0$ and $x = 1$.

Theorem 1.3 The area A of the region bounded by two curves $y = f(x)$ and $y = g(x)$ and the lines $x = a$ and $x = b$, where f and g are continuous on $[a, b]$ and $f(x) \geq g(x)$ for all x in $[a, b]$ is:

$$\int_a^b [f(x) - g(x)] dx$$

1.1.2 With respect to y 

Example 1.4 Find the area in the shaded region above.

Theorem 1.5 If a region is bounded by curves with equations $x = f(y)$, $x = g(y)$, $y = c$, and $y = d$, where f and g are continuous and $f(y) \geq g(y)$ for all y in $[c, d]$, then the area of this region is given:

$$\int_c^d [f(y) - g(y)] dy.$$

Example 1.6 Find the area in the first quadrant bounded by the curves $y = x$, $y = 4x$, $xy = 16$. *Hint: Sketch! You may need to split this into more than one integral.*