Math 1300: Calculus I **Fall 2020**

Lecture: Section 5.4: Fundamental Theorem of Calculus

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Today's Goal: Functions defined by integrals.

Logistics: We are starting this on Friday, finishing it on Monday, and then you've got quizzes (multiple!) Tuesday. They are cumulative, and will include everything through 5.4.

Warm-Up 1.1 *(Accumulation Function Worksheet Problem #2)*

1.1 Part II

We have seen the Evaluation Theorem, which is part II of the Fundamental Theorem of Calculus:

Theorem 1.2 *If $f(x)$ is continuous on $[a, b]$, and $F(x)$ is any antiderivative of $f(x)$ (i.e., $F'(x) = f(x)$), then:*

$$\int_a^b f(x)dx = F(b) - F(a).$$

Example 1.3 *Evaluate the integral:*

$$\int_0^{\pi/3} \frac{\sin(2x)}{\cos(x)} dx$$

1.2 Part I

In our worksheet, we saw that some functions can be defined by integrals. For example:

$$f(x) = \int_0^x (t^2 - 4t + 1)dt.$$

(a) It may be useful to find the general expression for an antiderivative of that function:

$$\int (t^2 - 4t + 1)dt =$$

(b) $f(2) =$

(c) $f(-1) =$

This motivates the Fundamental Theorem of Calculus Part I

Theorem 1.4 (Fundamental Theorem of Calculus Part I) *If $f(x)$ is continuous on $[a, b]$, then the function $g(x)$ defined*

$$g(x) = \int_a^x f(t)dt$$

for $a \leq x \leq b$ is an antiderivative of $f(x)$. In particular,

$$g'(x) = f(x).$$

Example 1.5 *Find the following derivative:*

$$\frac{d}{dx} \int_2^x \frac{\sin(4t-1)}{t^3 + 2t} dt$$

1.2.1 Proof

To prove this theorem, we will make the assumption that f has an antiderivative and apply the Evaluation Theorem:

$$\frac{d}{dx}\int_a^x f(t)dt =$$

Let's try another couple of examples. Evaluate the following derivatives:

Example 1.6 $\frac{d}{dx}\int_3^x e^{3t^2-4t}dt$

Example 1.7 $\frac{d}{dx}\int_x^1 \frac{1}{r^3-1}dr$

1.2.2 Chain Rule with FTC Part I

While FTC Part I is pretty simple, it can become more complicated if we combine it with the chain rule. For example, how would we find the following derivative:

$$\frac{d}{dx}\int_{3}^{g(x)} f(t)dt =$$

Try to think it through, using the proof assuming the existence of an antiderivative again:

1.2.3 Examples

Find the following derivatives

1. $\frac{d}{dt}\int_0^{t^2}\sqrt{y-y^3}dy =$

2. $\frac{d}{dx}\int_{3x^2-3x}^{1}\tan(t-4)dt =$

3. $\frac{d}{dx}\int_{2x}^{3x^2}\log_3(t^4+t)dt =$

4. $\frac{d}{dy}\int_{y^3}^{y^3-y}\frac{t^3}{t^2-1}dt =$