Math 1300: Calculus I **Fall 2020**

Lecture: Section 5.2: Definite Integrals

Lecturer: Sarah Arpin

Today's Goal: Define the definite integral; relate to the area problems we've been working on.

Logistics: We will start this on Wednesday and need to finish it on the following Monday. There is an activity on Friday. And a check-in on MONDAY this time around! As well as the following Friday (so Monday and Friday of the same week).

Warm-Up 1.1 *What is the area under the curve $y = x + 1$ between $x = 0$ and $x = 2$? Hint: Draw a picture to help - you can use geometric area formulas!*

(A) 1

(B) 2

(C) $\frac{3}{2}$

(D) 4

(E) None of the above.

Last section, we used rectangles to approximate areas, and limits to find them precisely. We will do a little review of this by discussing Midpoint Rule. Then, we will introduce a new notation for these precise areas, discuss when this object exists, and develop a new way to find these values.

1.1 Midpoint Rule

When we are computing sums of rectangle areas to estimate an area under the curve (computing a **Riemann Sum**, we have many options for choosing the height of the curve. Midpoint rule describes one such choice: using the midpoints of the intervals defining the rectangle widths.

Example 1.2 *Use midpoint rule with* $n = 5$ *to approximate the area under the curve* $y = \frac{2}{x-1}$ *between* $x = 1$ *and* $x = 3$.

1.2 New Notation for Area

The area under the curve $f(x)$ on the interval $[a, b]$ is computed by a limit of a sum of rectangle areas (a **Riemann sum**):

We introduce a new notation, using a symbol called an **integral:**

1.3 Evaluating Integrals as Limits of Riemann Sums

Example 1.3 *Evaluate* $\int_0^3 (x^3 - 2x)dx$ *using a limit of Riemann sums.*

Example 1.4 *Set up an expression for* $\int_1^2 2^x dx$ *as a limit of sums. Do not evaluate.*

We can also use the geometric interpretation of an integral in order to evaluate that integral. A few common examples of this:

Example 1.5 *Use geometry to evaluate:*

$$\int_{-2}^{2} \sqrt{4 - x^2} dx$$

Example 1.6 *Express the area under the curve* $y = 2x - 4$ *between* $x = 2$ *and* $x = 4$ *as an integral. Draw this region, and evaluate the integral geometrically.*

1.4 Properties of the integral

(1) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(2) $\int_a^a f(x)dx =$

(3) $\int_a^b c dx =$

(4) $\int_a^b cf(x)dx =$

(5) $\int_a^b [f(x) \pm g(x)]dx =$

(6) $\int_a^b f(x)dx + \int_b^c f(x)dx =$

(7) If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)dx \geq 0$

(8) If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$.

(9) If $L \leq f(x) \leq M$ for $a \leq x \leq b$, then $L(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$.