

Lecture: Section 4.8: Antiderivatives

*Lecturer: Sarah Arpin***Today's Goal: Learn about antiderivatives!**

Logistics: We will start and finish this section on Wednesday. Don't forget Check-In 14 on Friday! Review sections 4.6 and 4.8 (this section!).

Warm-Up 1.1 Find two positive numbers whose sum is 300 and whose product is a maximum.

(A) 150, 150

(B) 100, 200

(C) 50, 250

(D) 300, 300

(E) None of the above

1.1 Antiderivatives

1.1.1 Terminology

Find the derivative of $f(x) = x^3 - 2x + 1$:

$$f'(x) =$$

Let's just name this new function $g(x)$:

$$g(x) =$$

Since the derivative of $f(x)$ is equal to $g(x)$, we say that $f(x)$ is an **antiderivative** of $g(x)$.

Question: Are there other anti-derivatives of $g(x)$?

Theorem 1.2 If $F(x)$ is an antiderivative of $f(x)$, then so is $F(x) + C$, for any real number C .

Example 1.3 Find all functions $f(x)$ that satisfy $f'(x) = \frac{2}{1+x^2} - e^{-x}$.

Example 1.4 Find $f(x)$ if $f'(x) = 3x^2 - 4x + 5$ and $f(-1) = 2$.

Example 1.5 Find $g(t)$ if $g''(t) = 2e^t + 3\sin(t)$, $g(0) = 0$, and $g(\pi) = 0$.

Recall now and always that the acceleration due to gravity is $-9.8m/s^2$, or equivalently $-32ft/s^2$.

Example 1.6 *A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground?*

Example 1.7 *A stone was dropped off a cliff and hit the ground with a speed of 120 ft/s. What is the height of the cliff?*