Math 1300: Calculus I **Fall 2020**

Lecture: Section 4.3: Derivatives and Curve Sketching

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Today's Goal: Discuss the mean value theorem and how to sketch the graph of any function.

Logistics: We will start this section with an activity Tuesday and finish Wednesday. There is a check-in Friday, it's set to cover the topics we are covering this week: linear approximation, min/max problems, and extreme value theorem (Sections 3.9 & 4.2).

Warm-Up 1.1 *What are the critical numbers of the function* $f(x) = x^3 - 12x$*?*

(A) $-2, 0, 2$

(B) $-4, 4$

(C) $-2, 2$

(D) $-4, 0, 4$

(E) None of the above.

Warm-Up 1.2 *Use logarithmic differentiation to find* y' *for* $y = (\sin(x))^{x^2}$.

(A) $y' = (\sin x)^{x^2}(\frac{2x\cos(x)}{\sin x})$

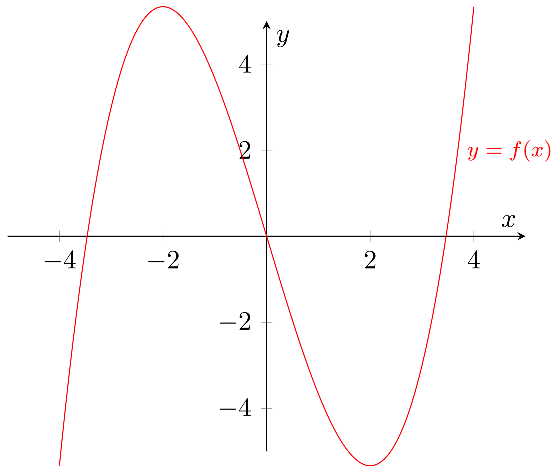
(B) $y' = -(\sin x)^{x^2}(\frac{2x\cos(x)}{\sin x})$

(C) $y' = (\sin x)^{x^2}(2x\ln(\sin x) - \frac{x^2\cos(x)}{\sin(x)})$

(D) $y' = (\sin x)^{x^2}(2x\ln(\sin x) + \frac{x^2\cos(x)}{\sin(x)})$

(E) None of the above

1.1 The Mean Value Theorem



Theorem 1.3 (The Mean Value Theorem) *If is a differentiable function on the interval* $[a, b]$*, then there exists a number* c *in* (a, b) *such that*

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Remark 1.4 *When the slope of the secant line is equal to 0, this is called **Rolle's Theorem**.*

Example 1.5 *Use the Mean Value Theorem to show that $f'(c) = 0$ for some $c \in (-1, 3)$, where $f(x) = x^2 - 2x - 8$.*

Example 1.6 *Suppose we know that $f(x)$ is continuous and differentiable on $[-7, 0]$ and that $f(-7) = -3$ and that $f'(x) \leq 2$. What is the largest possible value for $f(0)$?*

1.2 Sketching Techniques

1.2.1 Increasing/Decreasing

- If $f'(x) > 0$ on an interval, then...
- If $f'(x) < 0$ on an interval, then...

1.2.2 Local Minima and Maxima

Theorem 1.7 (First Derivative Test) *Suppose that c is a* ***critical number*** *of $f(x)$ (recall: this means c is in the domain of f and either $f'(c) = 0$ or $f'(c)$ DNE). Then:*

- *If f' changes from positive to negative at c, then...*
- *If f' changes from negative to positive at c, then...*
- *If f' does not change sign at c, then...*

1.2.3 Concavity

A function is called **concave upward** on an interval if (equivalently)...

- f' is...
- f'' is...

A function is called **concave down** on an interval if (equivalently)...

- f' is...
- f'' is...

Remember, $f(x)$ has an **inflection point** at $x = c$ if...

Theorem 1.8 (Second Derivative Test) *Suppose $f(x)$ is continuous near c.*

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a...

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a...

Example 1.9 $f(x) = 4x^3 + 3x^2 - 6x + 1$

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the local maximum and minimum values of f.

(c) Find the intervals of concavity and the inflection points.

Example 1.10 *From the textbook:*

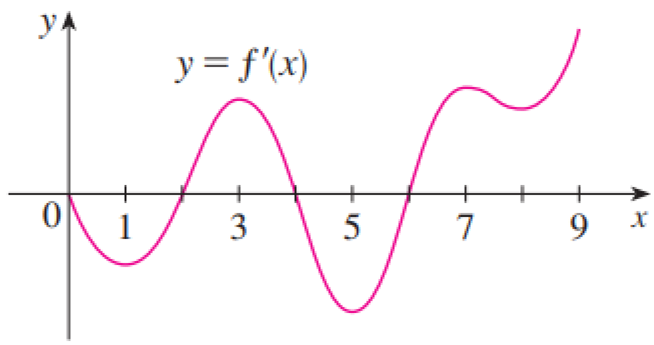
6. The graph of the first derivative f' of a function f is shown.

(a) On what intervals is f increasing? Explain.

(b) At what values of x does f have a local maximum or minimum? Explain.

(c) On what intervals is f concave upward or concave downward? Explain.

(d) What are the x-coordinates of the inflection points of f? Why?



Example 1.11 $f(x) = \frac{x^2}{(x-2)^2}$

(a) Find the vertical and horizontal asymptotes of $f(x)$.

(b) Find the intervals of increase or decrease.

(c) Find the local maximum and local minimum values.

(d) Find the intervals of concavity and inflection points.

(e) Sketch the graph, using all of the information you just found.