Math 1300: Calculus I **Fall 2020**

Lecture: Section 3.8: Rates of Change

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Today's Goal: Learn about applications of the derivative to other sciences.

Logistics: We start this on Wednesday and finish it on Friday. There is a check-in on Friday!

Warm-Up 1.1 *True or False:* $\frac{d}{dx}(\arctan(x^2)) = \frac{2x}{1+x^4}$.

Big Picture:

$$\text{Average Rate of Change of } f \text{ between time } x \text{ and } x+\Delta x = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$\text{Instantaneous rate of change of } f \text{ at time } x = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

1.1 Physics

1.1.1 Motion

Example 1.2 *The position of a particle at time t (seconds) is given $s(t) = \frac{1}{3}t^3 - t$. Draw a diagram to illustrate the motion of the particle. Find the total distance traveled by the particle in the first 8 sections. When is the particle speeding up? When is it slowing down?*

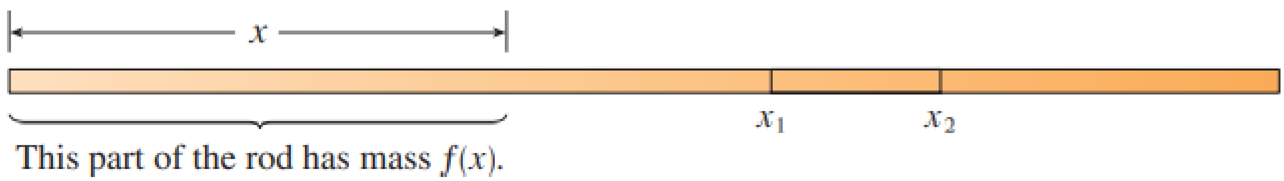
1.1.2 Linear Density

Linear density is defined as mass per unit length, and is usually denotes ρ. Think of a length of wire where the density of the wire changes between end to end.

When the mass is constant, this is easy to calculate: $\rho = \text{mass/length}$. However, if mass is changing we must use a formula:

$$\rho(x) = \lim_{\Delta x \to 0} \frac{m(x + \Delta x) - m(x)}{\Delta x} = m'(x)$$

Note that this is the derivative with respect to length (distance from left end of rod).



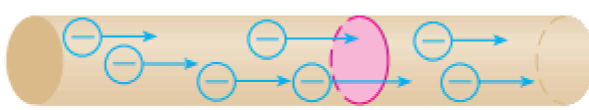
Example 1.3 *The mass of the part of a metal rod that lies between its left end and a point x meters to the right is $4x^3$ kg. Find the linear density when $x = 2$ meters.*

1.1.3 Current is the Derivative of Charge

Average current is given by:

$$\text{average current} = \frac{\Delta Q}{\Delta t}$$

where ΔQ is the amount of charge that passes through a fixed cross section of a rod over a period of time (Δt time).



1.2 Chemistry

1.2.1 Rate of Reaction

The concentration of a reactant A is the number of moles per liter and is denoted by [A]. The concentration varies during a reaction, leading to the definition:

$$\text{Average rate of reaction} = \frac{\Delta[A]}{\Delta t}$$

1.2.2 Compressibility

$$\text{Isothermal Compressibility} = \frac{-1}{V}\frac{dV}{dP},$$

where P is pressure and V is volume.

1.3 Biology

1.3.1 Population Growth

In the past, you have probably encountered **exponential growth**. If a population follows exponential growth, then the population at time t can be expressed:

$$P(t) = Ae^{kt}$$

where A is the population at time $t = 0$, and k is a growth constant depending on the population.

Differentiate to get the population growth rate:

$$P'(t) =$$

Example 1.4 *The number of yeast cells in a laboratory culture increases rapidly initially but levels off eventually. The population is modeled by the function*

$$n = f(t) = \frac{a}{1 + be^{-0.7t}}$$

where t is measured in hours. At time $t = 0$ the population is 20 cells and is increasing at a rate of 12 cells/hour. Find the values of a and b. According to this model, what happens to the yeast population in the long run.

1.4 Economics

1.4.1 Marginal Cost

The concept of average cost per unit is an important concept in production. If $C(x)$ is the total cost a company incurs producing x units of their products, then we can find the average rate of change of cost:

$$\frac{C(x+\Delta x)-C(x)}{\Delta x}$$

Taking the limit as $\Delta x \to 0$, we obtain an instantaneous rate of change of cost, which is referred to as the **marginal cost**:

$$\text{marginal cost} = \frac{dC}{dx}$$

Example 1.5 *The cost function for production of a commodity is* $C(x) = 339 + 25x - 0.09x^2 + 0.0004x^3$.
Find and interpret $C'(100)$.
Compare $C'(100)$ *with the cost of producing the 101st item.*