Math 1300: Calculus I **Fall 2020**

Lecture: Section 3.5: Implicit Differentiation

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Today's Goal: Find derivatives when a function isn't given as just $f(x) = \cdots$

Logistics: We will start and finish this section today, Friday. We also have a check-in in the last five minutes.

Warm-Up 1.1 *Find $f'(\pi)$ for $f(x) = (\sin^2(x) - x) \cdot (\cos(x) + 2)$.*

(A) 0

(B) 1

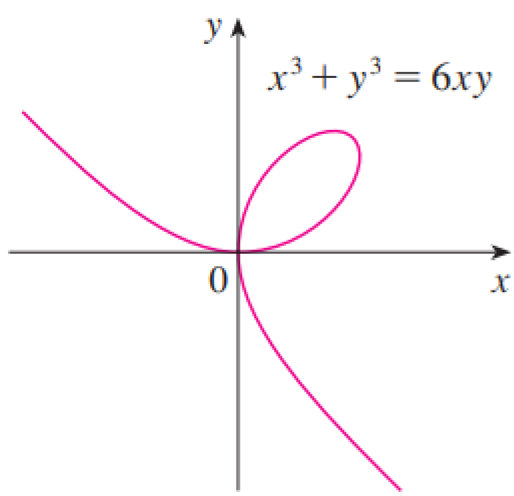
(C) -1

(D) $1 - \pi$

(E) *None of the above*

1.1 Implicit Differentiation

So far, we have been given an **explicit** function $f(x)$ or y wjere one side of the equation is written totally in terms of x. For example, we see things like $f(x) = x^2 - 2$, not ones with x's and y's on both sides of the equation, like $x^3 + y^3 = 6xy$, for example.



However, functions like $x^3 + y^3 = 6xy$ are important (this one is called *the Folium of Descartes*), and we would still like to be able to find slopes of tangent lines!

Key technique: Think of *applying the derivative* to both sides of an equation. For example, if we start with a function we know:

$$y = x^2 + 1$$

we can apply the derivative to both sides of the equation as such:

and simplify using our knowledge of derivatives to get an expression for $\frac{dy}{dx}$:

Apply this same technique to an implicit equation, just be sure to treat y as a function of x, instead of just another variable. Let's start with the folium of Descartes:

$$x^3 + y^3 = 6xy$$

Begin by *applying the derivative* to both sides of the equation, and we'll see how to interpret this as we go:

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(6xy)$$

Example 1.2 (An example we can visualize!) *(a) Find* $\frac{dy}{dx}$ *for* $x^2 + y^2 = 25$.

(b) Use the expression in part (a) to get the equation of a tangent line to the graph at the point $(-3, 4)$.

Example 1.3 *Find* y' *if* $\sin(x - y) = \frac{\cos(x)}{x^2+y^2+1}$

Example 1.4 *Find* y'' *if* $\sqrt[4]{x} - \sqrt[4]{y} = 12$.

Example 1.5 *Find equations of both the tangent lines to the ellipse* $x^2 + 4y^2 = 36$ *that pass through the point* $(12, 3)$.