Math 1300: Calculus I **Fall 2020**

Lecture: Section 3.2 The Product and Quotient Rules

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Today's Goal: Deal with derivatives of functions $f(x)g(x), \frac{f(x)}{g(x)}$

Logistics: We will start this on a Wednesday and finish on a Friday. Friday will be a check-in!

Warm-Up 1.1 *True or False:* $\frac{d}{dx}e^x = xe^{x-1}$

1.1 Product Rule

We want to be able to take the derivative of products of functions that we can't distribute, for example:

$$h(x) = e^x\sqrt{5x}$$

is the product of the two functions:
Our previous rules do not apply! We need a new rule! Let's go back to the definition of derivative and see if we get anywhere:

$$\frac{d}{dx}(f(x)g(x)) = \lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

A cool trick that is frequently helpful in math is to add and subtract a term, and see if that helps:

$$\frac{d}{dx}(f(x)g(x)) = \lim_{h\to 0}\frac{f(x+h)g(x+h) + f(x+h)g(x) - f(x+h)g(x) - f(x)g(x)}{h}$$

Definition 1.2 (Product Rule) *The product rule is:*

$$\frac{d}{dx}(f(x)g(x)) =$$

Example 1.3 *Find the derivatives of the following functions:*

(1) $h(x) = e^x\sqrt{5x}$

(2) $f(x) = \frac{\sqrt{2x}}{x^3}$ *Hint: Re-write the fraction as a product*

(3) $f(x) = x^2e^x + 3x - 1$

(4) $g(x) = (x^3 - 1)(e^x + \sqrt[4]{x^3})$

1.2 Quotient Rule

We also need a rule for taking the derivative of functions of the form $\frac{f(x)}{g(x)}$. Here is the rule:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) = f(x)g'(x)}{[g(x)]^2}$$

Example 1.4 *Find the derivatives of the following functions:*

1. $f(x) = \frac{3x+1}{4x^2+2}$

2. $g(t) = \frac{a+b}{c+2e^k}$, *for* a, b, c *real numbers*

3. $h(x) = \frac{1-xe^x}{2x+e^x}$

Example 1.5 *Suppose we have the following information about the differentiable functions f and g:* $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, *and* $g'(5) = 7$.
Find the following values:

1. $(fg)'(5)$

2. $h'(5)$ *for* $h(x) = 5f(x) - 4g(x)$

3. $h'(5)$ *for* $h(x) = \frac{g(x)}{1+f(x)}$

4. $h'(5)$ *for* $h(x) = \frac{g(x)f(x)+1}{g(x)}$