

Lecture 8: Section 2.7: The Derivative as a Function

*Lecturer: Sarah Arpin***Today's Goal:**

Logistics: Evening quiz this Tuesday! Set an alarm on your phone.

Speaking of phones...let's try to focus as much as possible during lecture. Take a minute - physically take your phone to the opposite corner of the room (if you are able), and turn on your camera (if you are able)!

It will help you say present.

Warm-Up 8.1 *What is the slope of the tangent line to $f(x) = x^2 + 1$ at the point where $x = 1$? HINT: $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ is the definition of the slope of the tangent line/instantaneous velocity of a function.*

8.1 Definition and Alternate Definition of Derivative

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

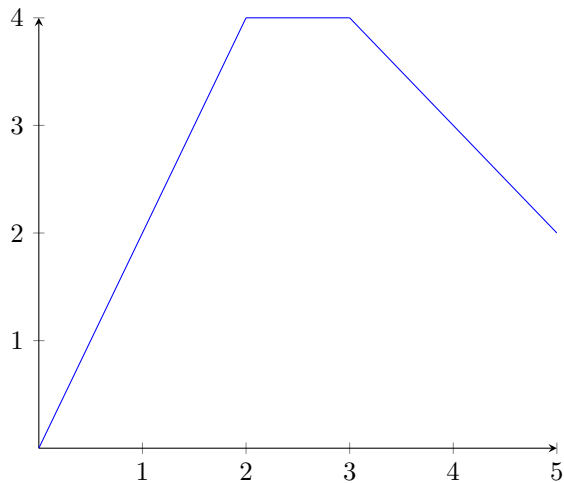
What about $f'(x)$?

8.2 Position and Velocity

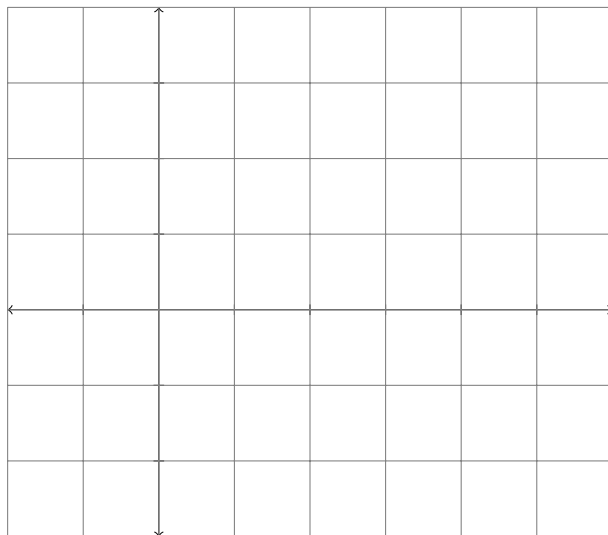
Suppose $s(t)$ gives the position of a particle at time t .

- $\frac{s(b) - s(a)}{b - a}$ = slope of secant line connecting the point $(a, s(a))$ and $(b, s(b))$ = average velocity of particle between times $t = a$ and $t = b$.
- $\lim_{\substack{h \rightarrow 0 \\ t = a}} \frac{s(a+h) - s(a)}{h}$ = slope of tangent line to $s(t)$ at $t = a$ = instantaneous rate of change of particle at $t = a$.

Sometimes we can read this information off of a graph. The following graph depicts the position $y = f(x)$ of a particle at time x :



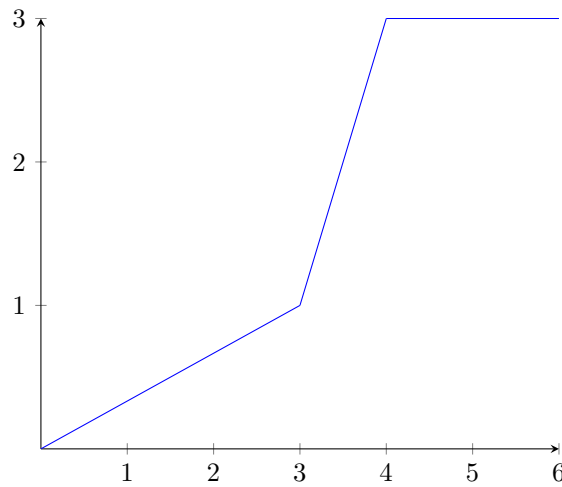
Now, let's graph the velocity:



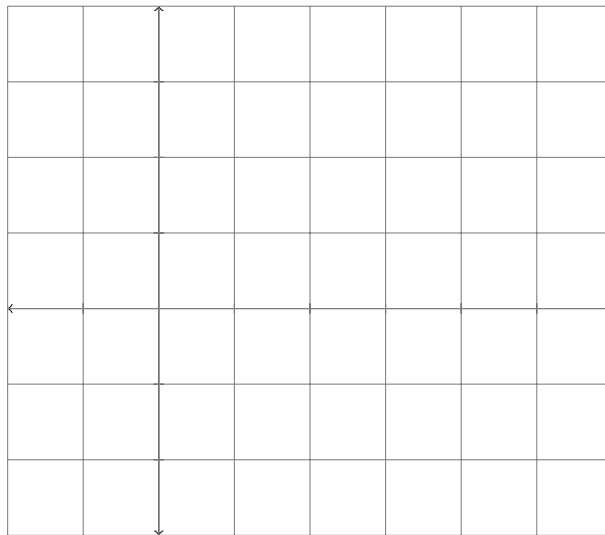
The graph we have drawn depicts the derivative of $y = f(x)$ as a function. We denote this function:

$$f'(x) \text{ or } \frac{dy}{dx}$$

Example 8.2 Suppose this graph depicts the position $y = f(x)$ of some particle at time x :



Now, let's graph the velocity:



Example 8.3 Does the expression

$$\lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h}$$

represent the derivative of some function? If so, what function?

Example 8.4 Does the expression

$$\lim_{h \rightarrow 0} \frac{\sin(h) - \sin(0)}{h}$$

represent the derivative of some function? If so, what function?

