MATH 6140: Second midterm examination. Wednesday, 31 March 2021.

Put **your name** on each answer sheet. Answer **both** questions.

Justify all your answers in full.

Formula sheets, calculators, notes and books are not permitted.

1. Let F be a field. Recall that $SL_3(F)$ is the set of all 3×3 matrices over F of determinant 1.

(i) Show that any matrix $A \in SL_3(\mathbb{C})$ that satisfies $A^3 = I$ is diagonalizable. Find the four possible Jordan Canonical Forms for such an A, and show that your list is complete.

(ii) For each of the four similarity classes arising from (i), and working over $\mathbb{C}$ in each case, find the associated (a) elementary divisors, (b) invariant factors, (c) Rational Canonical Form, (d) minimal polynomial, and (e) characteristic polynomial.

(iii) Show that two of the four matrices you listed in (i) above are each similar (over $\mathbb{C}$) to a matrix in $SL_3(\mathbb{Q})$, but that the other two are not.

2. Let $\mathbb{F}_3$ be the field with three elements, and let $A \in SL_3(\mathbb{F}_3)$ be a matrix that satisfies $A^3 = I$. Show that there are three similarity classes of such matrices, and for each similarity class, find the associated (a) elementary divisors, (b) invariant factors, (c) Jordan Canonical Form, (d) Rational Canonical Form, (e) minimal polynomial and (f) characteristic polynomial.