MATH 4140: Assignment 14

Chapter 22

1. Let G be a nonabelian group of order 39.
(i) Use Theorem 22.11 to show that G has seven conjugacy classes.
(ii) By considering the commutator subgroup, G', show that G has a normal subgroup of order 13.

2. Prove that every group of order 33 is abelian.

3. Let G be a nonabelian group of order 12.
(i) Prove that G must have a normal subgroup of order 3 or 4.
(ii) Give an example of a nonabelian group of order 12 with a normal subgroup of order 3.
(iii) Give an example of a nonabelian group of order 12 with a normal subgroup of order 4.

4. Let G be a nonabelian group of order 56.
(i) Explain why G cannot have $1, 1, 1, 2, 7$ as its character degrees.
(ii) Show that G has at least four linear characters.
(iii) Prove that there are no simple groups of order 56.

5. Let G be the alternating group A_5, and let $g \in G$ be a 5-cycle. Recall that the entries in the character table of G are all real numbers, and that there is an irreducible character χ of G for which $\chi(g)$ is not an integer.
(i) Prove that g is conjugate to g^4 in G.
(ii) Prove that g^2 is conjugate to g^3 in G.
(iii) Use Theorem 22.16 to show that g is not conjugate to g^2 in G.